A study of trends of neutron skin thickness and proton radii of mirror nuclei

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Introduction

It has been suggested that the differences in the charge radii of mirror nuclei are proportional to the slope of the symmetry energy $L$ and to the neutron-skin thickness of neutron-rich nuclei [1]. The neutron skin of atomic nuclei affects the structure of neutron-rich nuclei, the equation of state of nucleonic matter and the size of neutron stars. The neutron skin of nuclei is an important fundamental property but its accurate measurement faces many challenges. Inspired by charge symmetry of nuclear forces the neutron skin of nucleus is related to the difference between the charge radii of corresponding mirror nuclei. The direct determination of neutron-skin thickness usually involves the precise measurement of the root-mean-square radii of both the charge and mass distributions [2].

Assuming perfect charge symmetry in case of mirror nuclei,

$$R_n(Z, N) = R_p(Z, N) \quad (1)$$

The neutron-skin thickness is given by

$$S_n(Z, N) = R_n(Z, N) - R_p(Z, N) \quad (2)$$

$$S_n(Z, N) = R_p(N, Z) - R_p(Z, N) \equiv \Delta R_p$$

Theoretical Formalism

A. Meson exchange model

The Lagrangian density for meson exchange model can be written as [3]:

$$\mathcal{L} = \sum_i \overline{\psi}_i (i \gamma_\mu \partial^\mu - m) \psi_i + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2$$

$$- \frac{1}{2} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \overline{R}_\mu R^{\mu}$$

$$+ \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - g_\sigma \overline{\psi} \psi \sigma$$

$$- g_\omega \overline{\psi} \gamma^\mu \rho_\mu \psi - g_\rho \overline{\psi} \gamma^\mu \gamma^\nu \rho_{\mu\nu} \psi - e \overline{\psi} \gamma^\mu \psi A^\mu \quad (3)$$

where the first term represent the Lagrangian of free nucleons. $m_\sigma$, $m_\omega$, $m_\rho$ represents the masses of $\sigma$, $\omega$, and $\rho$ mesons with corresponding coupling constants $g_\sigma$, $g_\omega$, $g_\rho$ for the mesons to the nucleons, respectively. $\Omega_{\mu\nu}$, $R^\mu$, $F_{\mu\nu}$ are field tensor of the vector fields $\omega$, $\rho$, and the photon. The coupling of $\sigma$ field and $\omega$ field to the nucleon field reads

$$g_i(\rho) = g_i(\rho_{\text{sat}}) f_i(x) \quad \text{for} \quad i = \sigma, \omega \quad (4)$$

with

$$f_i(x) = a_i + b_i \left( \frac{x + d_i}{x + d_i} \right), \quad (5)$$

with $x = \rho/\rho_{\text{sat}}$. Here, $\rho_{\text{sat}} (=0.152 \text{fm}^{-3})$ is the baryon density at saturation in symmetric nuclear matter.

For density dependence of $\rho$-meson coupling is given by

$$g_\rho(\rho) = g_\rho(\rho_{\text{sat}}) e^{-a_\rho(x-1)} \quad (6)$$

The present model is represented by the parameter sets DD-ME1 and DD-ME2 [3].

Results and Discussion

Fig. 1 presents the results for the neutron skin thickness of some selected nuclei in light
to medium mass region. The neutron skin thickness is plotted against the isospin asymmetry $\delta = (N - Z)/A$. It is clear from the figure that there is a strong linear relationship between the skin-thickness $\Delta R_{np}$ and isospin asymmetry $\delta$. This correlation was first derived in Ref. [4] from the liquid drop model and determined with mean-field method. We fit the data using linear regression which is shown in Fig. 1 as shaded pink band at 1σ, 2σ and 3σ confidence levels.

FIG. 1: Neutron skin thickness plotted against the isospin asymmetry using DD-ME1 and DD-ME2 interactions. The linear regression is printed in the bottom right. Select nuclei are labeled.

To establish a general equation for the neutron-skin, we can also use our results to establish a relationship between the proton radius of a nucleus, $R_p$, and the neutron radius of its mirror partner, $R_n^\text{mirror}$. The resulting linear relationship between $R_p$ and $R_n^\text{mirror}$ is shown in Fig. 2. The slope of this linear relation is just less than unity. This highlights the deviation from isospin symmetry, which is due to the Coulomb interaction. The near-equivalence between $R_p$ and $R_n^\text{mirror}$ indicates a direct relation between the neutron skin and the mirror-difference charge radii.

FIG. 2: The neutron radius plotted against the proton radius of the corresponding mirror nucleus. The linear regression is printed in the bottom right and 1σ, 2σ, and 3σ confidence levels are shown as overlapping red bands. Select nuclei are labeled.

The study of proton radii of mirror nuclei is useful because neutron distributions are very difficult to measure experimentally. So the proton radius can be used as a surrogate for neutron skin thickness. Also, the linear relationship between the $R_p$ and $R_n^\text{mirror}$ can be used to estimate neutron radii of rare isotopes which are not yet accessible experimentally.

References

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