Locating inner edge of the neutron star crust from stability of neutron star matter

Debasis Atta* and D.N. Basu†
Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Kolkata 700 064, INDIA

Introduction

One of the most important predictions of nuclear equation of state (EoS) is the location of the inner edge of neutron star crust. Knowledge of the properties of the crust plays an important role in understanding many astrophysical observations. The inner crust spans the region from the neutron drip point to the inner edge separating the solid crust from the homogeneous liquid core. While the neutron drip density is relatively well determined, the transition density ρ_t at the inner edge is still largely uncertain mainly because of limited knowledge on EoS, especially the density dependence of the symmetry energy, of neutron-rich nuclear matter. At the inner edge a phase transition occurs from the high-density homogeneous matter to the inhomogeneous one at lower densities. The transition density takes its critical value ρ_t when the uniform neutron-proton-electron matter (npe) becomes unstable with respect to the separation into two coexisting phases (one corresponding to nuclei, the other to a nucleonic sea). In the present work, stability of the β-equilibrated dense nuclear matter is analyzed with respect to the thermodynamic stability conditions. Based on the density dependent M3Y (DDM3Y) [1] effective nucleon-nucleon (NN) interaction, the effects of nuclear incompressibility on proton fraction in neutron stars and location of the inner edge of their crusts and core-crust transition density and pressure are investigated.

Stability of β-stable neutron star matter

The quantity $V_{thermal}$ which determines the thermodynamic instability region of neutron star matter at β-equilibrium is given by $V_{thermal} = -(\frac{\partial P}{\partial v})_\mu$ which in terms of ρ, ϵ and x_p can be written as:

$$V_{thermal} = \rho^2\Big[2\rho\frac{\partial\epsilon}{\partial\rho} + \rho^2\frac{\partial^2\epsilon}{\partial\rho^2} - \frac{(\rho\frac{\partial^2\epsilon}{\partial\rho\partial x_p})^2}{\frac{\partial^2\epsilon}{\partial x_p^2}}\Big] \quad (1)$$

where ρ, ϵ and x_p are the baryonic number density, energy/baryon and β-equilibrium proton fraction, respectively [1]. The intrinsic stability condition of a single phase for locally neutral matter under β-equilibrium is determined, thermodynamically, by the positivity of the $V_{thermal}$, under constant chemical potential. However, the limiting density that breaks these conditions will correspond to the core-crust (liquid-solid) phase transition. Thus transition density ρ_t (with corresponding pressure P_t and proton fraction $x_{p(t)}$) is determined at which $V_{thermal} = 0$ and goes to negative with decreasing density. The β-equilibrium proton fraction x_p is obtained by solving equation $\mu_p - \mu_n = \mu_e$ where $\mu_p - \mu_n = \frac{\partial\epsilon(\rho,x_p)}{\partial x_p}$ whereas $\mu_e = \sqrt{p_e^2c^2 + m_e^2c^4} \approx p_e c = \hbar k_f c = \hbar c(3\pi^2\rho x_p)^{1/3}$ where m_e and ρ_e being the rest mass and number density of electron, respectively, and from charge neutrality $\rho_e = \rho_p = \rho x_p$. Thus

$$\hbar c(3\pi^2\rho x_p)^{1/3} = -\frac{\partial\epsilon(\rho,x_p)}{\partial x_p} = +2\frac{\partial\epsilon}{\partial X}, \quad (2)$$

where isospin asymmetry $X = 1 - 2x_p$.

Calculations and Results

The stability of the β-equilibrated dense matter in neutron stars is investigated and the location of the inner edge of their crusts and core-crust transition density and pressure are determined using the DDM3Y effective NN interaction. The results for the transition density, pressure and proton fraction at the inner edge separating the liquid core from the

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[*] Electronic address: `datta@vecc.gov.in`
[†] Electronic address: `dnb@vecc.gov.in`

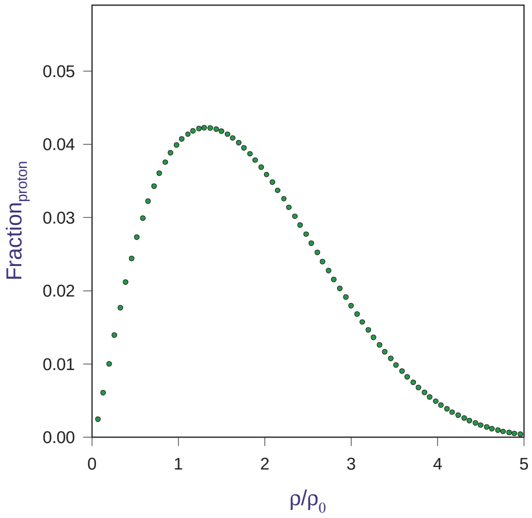

FIG. 1: The β equilibrium proton fraction obtained from calculations using DDM3Y interaction are plotted as functions of ρ/ρ_0.

TABLE I: Variations of the core-crust transition density, pressure and proton fraction for β stable neutron star matter, SNM incompressibility K_∞ and the isospin dependent part K_τ of the isobaric incompressibility with parameter n.

n	ρ_t	P$_t$	x$_{p(t)}$	K_∞	K_τ
	fm^{-3}	MeV/fm^3		MeV	MeV
Expt. values	- - - -	- - - -	- -	250-270	-370±120
1/6	0.0797	0.4134	0.0288	182.13	-293.42
1/3	0.0855	0.4520	0.0296	212.98	-332.16
1/2	0.0901	0.4801	0.0303	243.84	-370.65
2/3	0.0938	0.5006	0.0308	274.69	-408.97
1	0.0995	0.5264	0.0316	336.40	-485.28

solid crust of neutron stars are calculated and presented in Table-1 with parameter n which controls the nuclear matter incompressibility. The symmetric nuclear matter (SNM) incompressibility K_∞ and isospin dependent part K_τ of the isobaric incompressibility are also tabulated since for n=2/3 (symmetry energy = 30.71 MeV at saturation density and its slope L = 45.11 MeV), these are all in excellent agreement with the constraints recently extracted from the measured isotopic dependence of giant monopole resonances in even-A Sn isotopes, from the neutron skin thickness of nuclei and from analyses of the experimental data on isospin diffusion and isotopic scaling in intermediate energy heavy-ion collisions. It is worthwhile to mention here that the present EoS for n=2/3, provides maximum mass for the static case as 1.92 M$_\odot$ with radius 9.7 km and for the star rotating with Kepler's frequency it is 2.27 M$_\odot$ with equatorial radius 13.1 km [1] which reconcile with recent observations of massive compact stars ~2 M$_\odot$ [2, 3].

Summary and Conclusion

The effective NN interaction used in the present work, which is found to provide a unified description of elastic and inelastic scattering, various radioactivities and nuclear matter properties, also provides an excellent description of the β-equilibrated neutron star matter which is stiff enough at high densities to reconcile with the recent observations of the massive compact stars [1] while the corresponding symmetry energy is supersoft as preferred by the FOPI/GSI experimental data. The density, the pressure and the proton fraction at the inner edge separating the liquid core from the solid crust of the neutron stars determined to be ρ_t = 0.0938 fm^{-3}, P$_t$ = 0.5006 MeV fm^{-3} and x$_{p(t)}$ = 0.0308, respectively, are also in close agreement with other theoretical calculations [4] corresponding to high nuclear incompressibility and with those obtained using SLy4 interaction [5].

References

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