Thermodynamics of QCD matter at finite volume

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The strongly interacting matter has a rich phase structure at finite temperatures/densities. In the experiments these phases may be produced by ultra relativistic heavy ion collision (HIC) which are being carried out at CERN and BNL. The volume of the system thus created would depend on the nature of the colliding nuclei, the center of mass energy ($\sqrt{s}$) and the centrality of collision. The thermodynamics and the phase structure should depend on the size of the system (R). Here we would like to study the volume dependence of the thermodynamics of strongly interacting matter using an effective model of QCD namely PNJL model.

The PNJL model has been discussed in detail in the literature [1, 2]. To study the finite volume effects on the thermodynamic properties we begin with the thermodynamic potential Ω [1–3]. The saddle point of Ω gives the temperature/density dependence of the fields. For all R, at zero baryon density, we found that the system has a smooth crossover rather than a real phase transition. The crossover temperature (T_c) is identified to be the point of inflection of quark condensate and Polyakov loop with temperature. The results for T_c at different R are shown in table I.

R (fm)	2	2.5	3	5	∞
T_c (MeV)	167	171	180	184	186

TABLE I: T_c for different system sizes.

Table I shows that T_c has a strong dependence on system size. It varies from 167 MeV to 186 MeV meaning a change of about 10%.

We now study the situation at non-zero quark chemical potential μ_q. For infinite volume the phase transition is of first order at sufficiently high μ_q. At some smaller μ_q, the first order transition ends at a critical end point (CEP) where the transition is second order. At even smaller μ_q we have only a crossover. As the volume of the system is lowered we find the phase transition characteristics fade away. In Fig. 1 the phase diagram as a function of system size is shown. The dotted line corresponds to a crossover and the solid line corresponds to a first order transition. The black point corresponds to the CEP. Note that the CEP gradually shifts towards higher μ_q and lower T and finally disappears as the volume is reduced. This is an encouraging fact for the

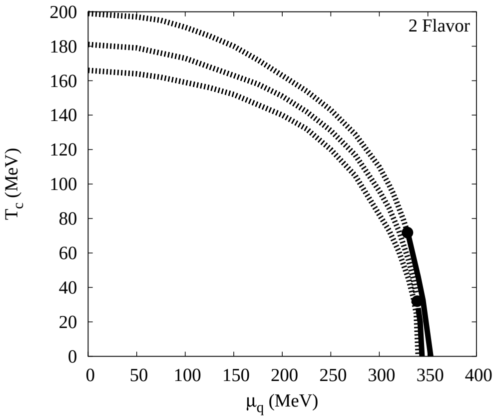

FIG. 1: Phase diagram for different system sizes. The inside curve is for $R = 2\ fm$, the next curve is for 2.5 fm and the outermost curve is for ∞.

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CEP search in HIC experiments. To attain such high densities one needs to collide the ions at low $\sqrt{s}$, which means the temperature attained is lower. As the experiments would produce finite system volumes this may lead to the location of the respective CEP possible [3]. The location of CEP for different volumes is collected in table II.

R (fm)	2	2.5	3	5	∞
T_c, (MeV)	No	32	52	69	72
μ_{q_c} (MeV)	CEP	339	335	330	329

TABLE II: CEP for different system sizes.

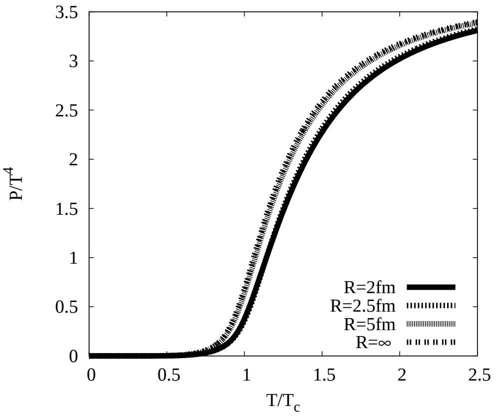

FIG. 2: Pressure as a function of temperature.

We now discuss some of the thermodynamic quantities. The pressure inside a volume V may be written as, $P(T,\mu_q) = -\frac{\partial(\Omega(T,\mu_q)V)}{\partial V}$. In Fig. 2 we plot the temperature dependence of scaled pressure (P/T^4). There is a significant change in scaled pressure for small system sizes. For example at T_c the P/T^4 for a system with $R = 2\,fm$ is almost half of that of an infinite system. As the temperature increases the difference slowly diminishes.

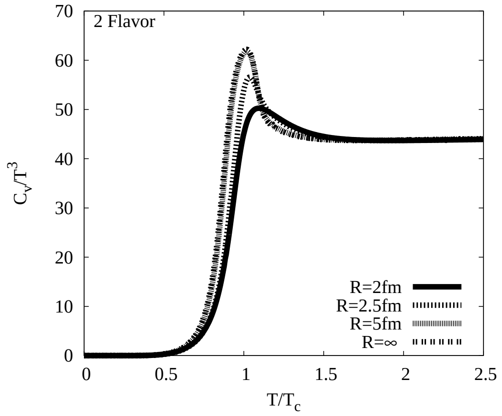

FIG. 3: (Color online) Variation of specific heat with temperature for different system sizes.

The specific heat at constant volume C_V is shown in Fig. 3. We find that with the change in volume, C_V changes prominently up to the temperature corresponding to the crossover region. For smaller volumes the specific heat is smaller indicating a higher rise in temperature for the same rise in energy density. The specific heat is also a measure of energy fluctuations in the system [3] which rise sharply near a phase transition.

To summarise we have studied the strongly interacting matter inside finite volume using PNJL model. Several interesting results were observed that can have important implications for heavy-ion collision experiments. Our major finding was that the spontaneously broken chiral symmetry may be restored at much lower temperatures in small volume. We also demonstrated a stronger possibility of finding the signatures of a critical end point in low energy experiments that intend to create high baryonic densities where the expected temperature is not too high.

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