Charge densities in transverse coordinate and impact parameter space

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1. Introduction

Study of form factors has been important in the field of hadron physics as they provide important information about internal structure of the nucleon[1]. Recently, experiments have measured and produced remarkably precise data for the electromagnetic form factors. The nucleon electromagnetic form factors have been used to study the distribution of charge and magnetization densities of quarks inside the nucleon. To get a more clear picture of the nucleon structure, one can fully parameterize the structure of the nucleon in terms of generalized parton distributions (GPDs) which are functions of longitudinal momentum x of the quark, the invariant momentum transfer t and the skewness parameter ζ which gives the fraction of the longitudinal momentum transfer to the nucleon. The Fourier transform of the GPDs w.r.t transverse momentum transfer has not only been used to study the nucleon structure in transverse impact parameter space but it also gives a tomographic picture of the distribution of quark charge densities inside the nucleon through the impact parameter dependent parton distribution function (ipdpdf) $q(x, b_\perp)$ for a quark of momentum fraction x located at a transverse position $b_\perp$ [2]. It has been shown that ipdpdf of the transversely localized states of the same helicity is the two-dimensional Fourier transform of the spin non-flip GPD H. Transverse charge density defines a way to analyze electromagnetic form factors of hadrons moving at infinite momentum with transverse distance $b_\perp$ from the transverse center of momentum. Charge densities have been discussed in impact parameter space for the polarized and transversely polarized nucleons, in finite radius approximation, in scalar diquark model using transverse coordinate space as well as using Sachs form factors. Transverse densities can also be useful in the study of the spatial distribution of the momentum P^+ and can be related with the Fourier transforms of the gravitational form factors $A(q^2)$ and $B(q^2)$. In addition to this, magnetization density can be evaluated to get information on the anomalous magnetic moment of the nucleon. Recently, transverse charge and magnetization densities have also been studied in the holographic QCD.

2. Charge densities in transverse coordinate space and impact parameter space

The LFWFs in transverse coordinate space $\tilde{\psi}(x, \vec{r}_\perp)$ can be obtained by Fourier transforming $\psi(x, \vec{k}_\perp)$ in the momentum space

$$\tilde{\psi}(x, \vec{r}_\perp) = \int \frac{d^2 k_\perp}{(2\pi)^2} e^{i\vec{k}_\perp \cdot \vec{r}_\perp} \psi(x, \vec{k}_\perp), \quad (1)$$

where $\vec{r}_\perp$ is the transverse coordinate. The LFWFs for the spin up state in the transverse coordinate space can now be expressed as

$$\begin{aligned}
\tilde{\psi}^{\uparrow}_{+\frac{1}{2}+1}(x, \vec{r}_\perp) &= i\frac{e}{8\pi}\sqrt{2}M^2(1-x)^{\frac{3}{2}} r_\perp \\
&\quad K_0(\sqrt{D} r_\perp), \\
\tilde{\psi}^{\uparrow}_{+\frac{1}{2}-1}(x, \vec{r}_\perp) &= -i\frac{e}{8\pi}\sqrt{2}M^2 x\sqrt{1-x}\, r_\perp \\
&\quad K_0(\sqrt{D} r_\perp), \\
\tilde{\psi}^{\uparrow}_{-\frac{1}{2}+1}(x, \vec{r}_\perp) &= -\frac{e}{4\pi}\sqrt{2}M^2(Mx - m) \\
&\quad (1-x)^{\frac{3}{2}} \frac{1}{\sqrt{D}} r_\perp K_1(\sqrt{D} r_\perp), \\
\tilde{\psi}^{\uparrow}_{-\frac{1}{2}-1}(x, \vec{r}_\perp) &= 0, \quad (2)
\end{aligned}$$

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whereas for spin down state we have

$$\begin{aligned}
\tilde{\psi}^{\downarrow}_{+\frac{1}{2}+1}(x,\vec{r}_\perp) &= 0, \\
\tilde{\psi}^{\downarrow}_{+\frac{1}{2}-1}(x,\vec{r}_\perp) &= -\sqrt{2}\frac{e}{4\pi}M^2(Mx-m) \\
&\quad (1-x)^{\frac{3}{2}}\frac{r_\perp}{\sqrt{D}}K_1(\sqrt{D}r_\perp), \\
\tilde{\psi}^{\downarrow}_{-\frac{1}{2}+1}(x,\vec{r}_\perp) &= i\frac{e}{8\pi}\sqrt{2}M^2x(1-x)^{\frac{1}{2}}r_\perp \\
&\quad K_0(\sqrt{D}r_\perp), \\
\tilde{\psi}^{\downarrow}_{-\frac{1}{2}-1}(x,\vec{r}_\perp) &= -i\frac{e}{8\pi}\sqrt{2}M^2r_\perp(1-x)^{\frac{1}{2}} \\
&\quad K_0(\sqrt{D}r_\perp), \qquad (3)
\end{aligned}$$

where K is the Bessel function of second kind. A precise relation between the nucleon density distribution $P(r_\perp)$ and charge distribution $\rho(b_\perp)$ can be obtained using elementary convolution theorems and Fourier transforms as

$$\rho(x,\vec{b}_\perp) = \frac{1}{(1-x)^2}P\left(x,\frac{\vec{b}_\perp}{-1+x}\right). \qquad (4)$$

In order to obtain the explicit contributions of the u and d quarks in the nucleon density distribution $P(r_\perp)$ and charge distribution $\rho(b_\perp)$, we have used the isospin symmetry by using $P_u = P_p + \frac{P_n}{2}$, $P_d = P_p + 2P_n$ and the same relations for charge density in impact parameter space.

3. Conclusions

These plots give the complete spatial information about the nucleon. It is clear from the plots that charge and density distributions increase with the increasing value of the impact parameter $b_\perp$ and the transverse coordinate $r_\perp$ respectively, reach a maxima and then start decreasing. It is important to mention here that since x is the momentum fraction of the active quark, at $x = 1$, the active quark carries all the momentum and the contribution from other partons is expected to be zero at this limit. A cursory look at distribution reveals that their is slightly difference between the impact parameter and transverse coordinate space. GPDs H and E have dependence on x, ζ and t. Here x is the longitudinal momentum fraction, ζ is the longitudinal momentum transfer and t is the invariant momentum transfer. We have fixed the $\zeta = 0$ which in results fixed the longitudinal momentum fraction x in the impact parameter ($b_\perp$) space and one can get the exact information about the charge distribution in impact parameter space. But in the case of transverse coordinate space, we have no such parameter like ζ, $r_\perp$ is a simple coordinate parameter directly obtained by taking Fourier transform with momentum $k_\perp$ thus loosing control over the longitudinal momentum fraction x which in results we get the slightly difference of charge distribution between the behaviour in both impact parameter and coordinate space i.e. one get contribution from longitudinal momentum too in transverse coordinate space.

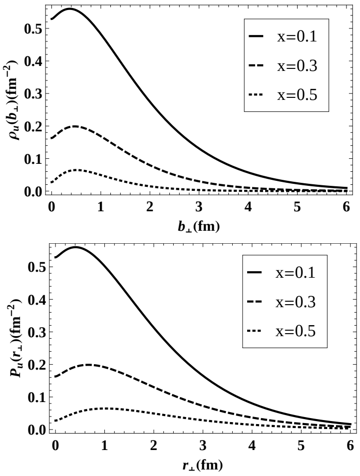

FIG. 1: Results for $\rho_u(x, b_\perp)$ (in fm^{-2})and $P_u(x, r_\perp)$ (in fm^{-2}) for the up quark in $b_\perp$ (in fm)and $r_\perp$ (in fm) space respectively.

References

[1] M. K. Jones *et al.*, Phys. Rev. Lett. **84** , 1398 (2000).

[2] M. Diehl, Eur. Phys. C **25**, 223 (2002).