Differential Equation Model both for B(E2)↑ and Excitation Energy E2 of Even-Even Nuclei

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Introduction

Existence of symmetry in nuclear dynamics explored in developing mass formulas such as the Garvey-Kelson [1] mass relations that connect masses of six neighboring nuclei most possibly led Ross and Bhaduri[2] in developing difference equations involving both B(E2)↑ and the corresponding excitation energy E2 of neighboring even-even nuclei. In this regard even more simpler difference equations connecting four neighboring even-even nuclei have been successfully derived by Patnaik et al.[3]. Just recently we[4] have succeeded in developing a differential equation model (DEM) for the B(E2)↑, according to which the B(E2)↑ value of a given even-even nucleus has been expressed in terms of its derivatives with respect to its neutron and proton numbers. Of late we realize that a similar differential equation could also be satisfied by its complementary physical quantity, namely the excitation energy E2. In general, both of them happen to reflect the deformations and nuclear shell structure of a given nucleus. Therefore in principle, the differential equation developed[4] earlier for B(E2)↑ could be generalized to include the quantity E2 also. It is needless to stress here that any relation in the form of a differential equation for any physical quantity is intrinsically sound enough to posses a good predictive ability. This philosophy has been well-demonstrated in case of B(E2)↑ predictions [4] just recently, and also over the years in the development [5–8] of the Infinite Nuclear Matter (INM) model of atomic nuclei. Therefore in the present work we first generalize the differential equation already developed for B(E2)↑ to include E2 and then examine its possible theoretical basis and experimental validity exclusively for E2.

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Generalization of the Differential Equation for B(E2)↑ and E2

The development[4] of the DEM model for the B(E2)↑ and presently for E2 owes its origin to the local energy differential equation in the INM model[5–8] of atomic nuclei. Physically the local energy η embodies all the characteristic properties of a given nucleus, mainly the shell and deformation, and has been explicitly shown [9] to carry the shell-structure. Therefore it is likely to have some characteristic correspondence with the properties of excited states of a given nucleus in general and in particular, the reduced transition probability B(E2)↑ and also for its complementary quantity E2. The already developed[4] Differential Equation for B(E2)↑ is given by

$$\frac{B(E2)(N,Z)}{A} = \frac{1}{2}[(1+\beta)\Big(\frac{\partial B(E2)}{\partial N}\Big)_Z + (1-\beta)\Big(\frac{\partial B(E2)}{\partial Z}\Big)_N]. \quad (1)$$

where N, Z and A refer to the neutron, proton and mass number of the given nucleus. β is the asymmetry parameter (N-Z)/A of the nucleus. Here we generalize the above equation to include both the physical variables B(E2)↑ and E2 under a common name C as

$$\frac{C(N,Z)}{A} = \frac{1}{2}[(1+\beta)\Big(\frac{\partial C}{\partial N}\Big)_Z + (1-\beta)\Big(\frac{\partial C}{\partial Z}\Big)_N]. \quad (2)$$

Its theoretical basis for E2 can be established from the emperical fact that these are slowly varying functions of N and Z and hence locally they can be approximated as linear functions of N and Z except,however,across the magic numbers and in regions where deformations drastically change.

Recursion Relation in B(E2)↑ and E2

For deriving recursion relations, we use the usual forward and backward definitions for the two derivatives occuring in the Eq. (2) as difference of the corresponding values of neighboring even-even nuclei. These are

$$C[N,Z] = \frac{N}{A-2}C[N-2,Z] + \frac{Z}{A-2}C[N,Z-2] \quad (3)$$

$$C[N,Z] = \frac{N}{A+2}C[N+2,Z] + \frac{Z}{A+2}C[N,Z+2]. \quad (4)$$

Thus depending on the availability of data, one can use either or both of these two relations to obtain the corresponding unknown values of neighboring nuclei for both the physical quantities B(E2)↑ and E2.

Numerical Test of the Recursion Relations for E2

For this we take as usual the adopted E2 data set of Raman et al. [10] in the above relations (3,4) and follow the procedure laid down by Raman et al. [11] in computing the deviations of the calculated E2 values from those of the experimental data in terms of the percentage errors. In general we find good agreement with experimental data. Apart from this study we have also find good agreement of our predictions with the latest experimental data of Pritychenko et al.[13] for even-even Cr,Fe,Ni and Zn nuclei near $N \sim Z \sim 28$.

Concluding Remarks

Thus the Differential Equation Model (DEM) developed earlier[4] for B(E2)↑ has been shown here to hold good even for the corresponding complementary quantity namely the excitation energies E2 of even-even nuclei. Accordingly both these physical quantities of a given nucleus can satisfy a common differential equation in which both of them individually can be expressed in terms of their derivatives with respect to the neutron and proton numbers. Then by exploiting definitions of the derivatives occurring in the differential equation, we could obtain recursion relations connecting in each case three neighboring even-even nuclei from lower to higher mass numbers and vice-versa, which are found to be highly useful for predicting unknown data.

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