Magnetic Moment of Delta Baryons with Extended Sea

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Introduction:

Hadronic physics revealed the details of nucleonic structure in terms of quarks and gluons. One of the well known resonances in nucleonic family is $\Delta(1232)$ but its magnetic dipole moment still lacks precise data due to unstable nature and very short lifetime. The measurement of magnetic moment Δ^{++} and Ω^{-} were reported long back[1-2]. The magnetic moment of Δ^{++} can be determined by $\pi^{+}p \rightarrow \pi^{+}p\gamma$ reactions whereas the determination of magnetic moment of Δ^{+} requires processes like radiative pion photoproduction. The most feasible range is for $\mu(\Delta^{+}) = 2.7^{+1.0}_{-1.3} \pm 1.5$ [3] as predicted experimentally. Therefore, many theoretical models like chiral quark model [4], soliton models [5] etc. are motivated to calculate the magnetic moments. Due to the instability of Δ-particles, the computation of magnetic moments by experimental and theoretical approaches are affected by error bars. In this work, we extend the non-relativistic quark model for delta baryons with an aim to include the sea quark-antiquark pairs with gluons. We analyze the effect of adding quark-gluon Fock states to the magnetic moment of delta resonances. The calculations involve framing of a general wave-function with due inclusion of sea-content keeping in mind the total antisymmetry of the wave-functions and C.G coefficients.

Construction of Wave-function:

Our aim is to write a wave-function suitable for all delta baryons by including possible combinations of sea quark-gluon Fock states. The sea quark-gluon Fock states can be $|gg\rangle$, $|q\bar{q}g\rangle$, $|q\bar{q}q\bar{q}\rangle$. For simplicity, initially all the sea quark-gluon Fock sates are assumed in a color singlet state. Thus, the wave-function is comprised of two parts, one is for q^3 part denoted as Ψ and it is accompanied by suitable combinations of sea denoted as H and G.

$$\Psi = \Psi(|\Phi\rangle|\chi\rangle|\psi\rangle|\xi\rangle$$

Where Φ, χ, ψ and ξ denote flavor, spin, color and space-time q^3 wave function. For the lowest-lying hadrons, quarks lie in S-wave and the space–time q^3 wave function. ξ is total symmetric under permutations of two quarks. Hence, the flavor-spin–color part should be total anti-symmetric under $q_i \leftrightarrow q_j$. The possible combinations for sea components can be H_0G_1, H_1G_1, H_2G_1. Thus, the elaborated form of spin-flavor-color wave-function of baryon decuplet along with sea-quark gluon Fock states is framed as:

$$|\Phi\rangle = \frac{1}{N}[a_1\Phi(10,\frac{3}{2},1)H_0G_1 + a_2[\sqrt{\frac{3}{5}}\Phi(10,\frac{3}{2},1)H_{1,0} - \sqrt{\frac{2}{5}}\Phi(10,\frac{1}{2},1)H_1,G_1]\otimes G_1 + a_3[\sqrt{\frac{2}{5}}\Phi(10,\frac{3}{2},1)\downarrow H_{2,2} - \sqrt{\frac{2}{5}}\Phi(10,\frac{3}{2},1)\uparrow H_{2,1}$$
$$+\sqrt{\frac{1}{5}}\Phi(10,\frac{3}{2},1)H_{2,0}]\otimes G_1]$$

Where N represents the normalized constant and a_1, a_2, a_3 are the co-efficients in the wave-function. The co-efficients associated with each term in the wave-function defines the probability associated with relevant Fock states. These coefficients can be best fitted using a fitting program in Mathematica. The fitting procedure involves the use of experimental data of magnetic moment of Δ^{++} and Δ^{0} as constraints to find the best fitted value. The program produces the several combinations but we chose the best fitted values of co-efficients come out to be: a_1 = 1.41; a_2 = 1.51; a_3 = 0.6. To calculate the magnetic moment of delta baryons, the operator $\widehat{o^i_f} = e^i/2m^i$ leads to the term:

$$\left\langle \Phi^{3/2\uparrow}\middle|\hat{O}\middle|\Phi^{3/2\uparrow}\right\rangle = \frac{1}{N^2}[a_1{}^2\left\langle(\Phi_{10}^{3/2\uparrow}\middle|\hat{O}\middle|\Phi_{10}^{3/2\uparrow}\right\rangle + a_2{}^2\left\langle(\Phi_{10}^{3/2\uparrow}\middle|\hat{O}\middle|\Phi_{10}^{3/2\uparrow}\right\rangle + a_3{}^2\left\langle(\Phi_{10}^{3/2\uparrow}\middle|\hat{O}\middle|\Phi_{10}^{3/2\uparrow}\right\rangle$$

Where $N^2 = a_1^2 + a_2^2 + a_3^2$. The first term is the conventional quark model result. In this, the term containing a_2 forms a vector sea while the term containing a_3 forms a tensor sea. The baryonic magnetic moment can thus be expressed in form of quark magnetic moment and a_1, a_2, a_3. The table given below shows the related expression and calculated values of magnetic moment in terms of μ_N and the comparison with other theoretical predictions.

Sr. No	Particle	Related Expression	Calculated Values	NRQM	XQM [6]
1.	Δ^{++}	$\Delta^{++} = a_1^2[3\mu_u] + a_2^2[\frac{11}{5}\mu_u] + a_3^2[\frac{7}{5}\mu_u]$	5.00	5.43	5.55
2.	Δ^{+}	$\Delta^{+} = a_1^2(2\mu_u + \mu_d) + a_2^2[\frac{3}{5}(2\mu_u + \mu_d) + \frac{2}{5}\mu_d] + \frac{a_3^2}{15}[16\mu_u + 7\mu_d]$	2.05	2.72	2.73
3.	Δ^{0}	$\Delta^{0} = a_1^2[(\mu_u + 2\mu_d)] + a_2^2[\frac{3}{5}(\mu_u + 2\mu_d) + \frac{2}{15}(\mu_u + 2\mu_d)] + a_3^2[\mu_u + \frac{2}{5}\mu_d]$	0	0	-0.09
4.	Δ^{-}	$\Delta^{-} = a_1^2[3\mu_d] + a_2^2[\frac{11}{5}\mu_d] + a_3^2[\frac{7}{5}\mu_d]$	-2.17	-2.72	2.91

Conclusion

In this we have constructed the baryon wave function with suitable quark-gluon Fock states for delta particles. In our study, the sea may be consisting of two gluons or a quark-antiquark pair along with a gluon. In our study, we constrain the sea with spin 0, 1, 2 and color singlet state for simplicity. We have calculated the magnetic moments for delta particles, after modification in valence quark wave function with due addition of sea component. It should be noted that our results and conclusions are subjected to the following points.

(1).The sea and the three quarks are considered in S-wave state (spin = 0, 1, 2).

(2).The sea is assumed to be flavorless and has been specified with total quantum numbers in terms of spin and color.

(3).Relativistic corrections have been neglected, although the inertial motion of light quarks in a baryon is expected to have relativistic motion.

(4). All the calculations are performed in baryon rest frame.

We are lacking in sufficient experimental data for magnetic moment of decuplet particles but we can check the feasibility of our approach by extending it to other spin 3/2 baryons. It would also be interesting to check it further for other baryonic properties. The extension of sea with color octet or decuplet states may be of further interest. Singh et al.[7] framed a statistical model to obtain the relevant probability of quark-gluon Fock state. The same model can also be applied to find desired information about the co-efficients of the wave-function.

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