Fractality in ^{28}Si-Ag/Br interaction at 14.5A GeV

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Multifractal structure of the pseudorapidity (η) distribution data on the singly charged particles produced in ^{28}Si-Ag/Br interaction at 14.5A GeV [1] is investigated. We calculate the frequncy moments G_q [2], and extract various parameters related to the multifractality there of. The experimental results are compared with a model calculation using the microscopic transport code – the Ultra-relativistic Quantum Molecular Dynamics (UrQMD) [3]. To check whether or not fractality, as observed in this analysis, is an outcome of the usual Bose-Einstein correlation (BEC) between identical bosons, we numerically incorporate the BEC into the UrQMD simulated events as an after burner on the basis of a charge reassignment algorithm [4]. The UrQMD data modified by the BEC are also used for comparison. Beside these three, a random number generated data set is used to eliminate the statistical noise.

Our result on G_q moments only for integer q are shown in FIG. 1 as a function of M – the η-space partition number. The diagrams show that the event averaged moment $\langle G_q \rangle$, scales with M following a power law. For each q the linear region of the $\ln \langle G_q \rangle$ *vs.* $\ln M$ plot is used to extract the mass exponent: $\tau(q) = -\partial(\ln \langle G_q \rangle)/\partial(\ln M)$. Knowing $\tau(q)$ one can now construct the multifractal spectral function: $f(\alpha_q) = q\alpha_q - \tau(q)$, where the Lipschitz-Holder exponent: $\alpha_q = \partial\tau(q)/\partial q$. In FIG. 2(a) we plot the τ_q and the α_q values against q, whereas the spectral function $f(\alpha_q)$ is shown in FIG. 2(b). Our results on multifractal parameters are very consistent with the predictions of the phenomenology of multifractality [2]. As for example, (i) $\tau(q)$ is not linear over the entire q interval – the slope of the $\tau(q)$ versus q curve decreases with increasing q, (ii) α_q decreases with increasing q, (iii) $f(\alpha_q)$ is concave downwards, is very stable, peaked around α_0, and the $f(\alpha_q) = \alpha_q$ line tangentially touches the spectrum at α_1. However, the experimental and the simulated results are not significantly different from each other, indicating the fact that the multifractality is present in the experiment as well as in the simulated data.

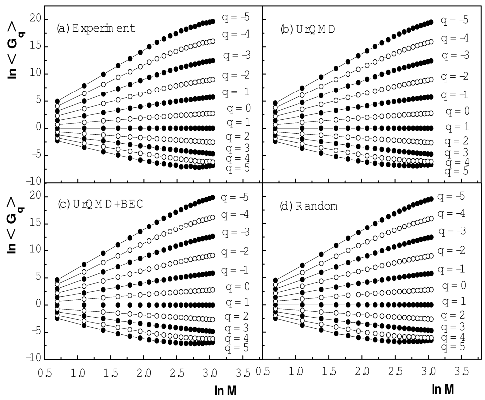

FIG. 1: $\ln \langle G_q \rangle$ as a function of $\ln M$.

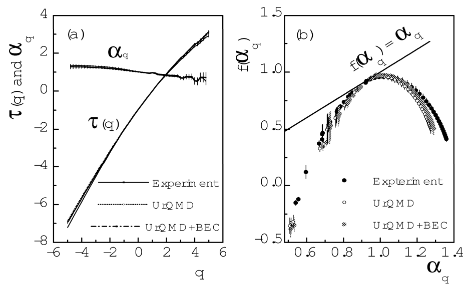

FIG. 2: (a) Event averaged mass exponents $\tau(q)$ and Lipschitz-Holder exponents α_q versus q plots. (b) Multifractal spectral function $f(\alpha_q)$.

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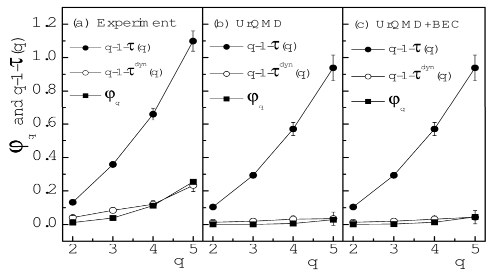

FIG. 3: Variation of ϕ_q, $q-1-\tau^{\rm dyn}(q)$ and $q-1-\tau(q)$ against q.

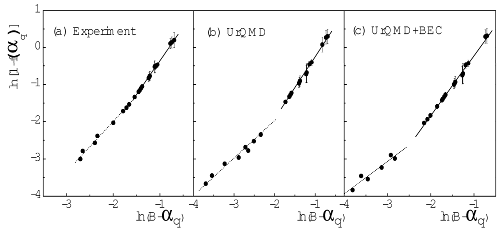

FIG. 4: $\ln[1-f(\alpha_q)]$ versus $\ln(B-\alpha_q)$ plots. The best fitted straight lines in different regions are shown.

The G_q moments themselves are not capable of eliminaing the statistical effect of fluctuations, and hence we take help of a random number generated event set. We denote the mass exponent of the random number generated data set by $\tau^{\rm sta}$, and the corresponding dynamical component by $\tau^{\rm dyn}(q)$. These two are related to each other by the relation: $\tau^{\rm dyn}(q) = \tau(q) - \tau^{\rm sta}(q) + q - 1$ [5]. There also exists a phenomenological relation between $\tau^{\rm dyn}(q)$ and the intermittency exponent ϕ_q, a measure of the strength of dynamical fluctuation as: $q-1-\tau^{\rm dyn}(q) \approx \phi_q$ [5]. In FIG. 3 we show this effect of eliminating the statistical contribution to the G_q moments. Whereas, $q-1-\tau(q)$ is way above ϕ_q, as soon as the statistical noise is subtracted the quantity $q-1-\tau^{\rm dyn}(q)$ comes down very close to the experimental ϕ_q [1]. As expected, UrQMD which is devoid of any correlation yields: $\phi_q \approx q-1-\tau^{\rm dyn}(q) \approx 0$, which indicates that the model shows multifractality but it is purely statistical in origin. On the other hand, compared to the experiment we find a very small but nonzero value of ϕ_q and $q-1-\tau^{\rm dyn}(q)$ in the UrQMD+BEC data. Hence the experiment contains certain jenuine correlation among produced particles.

Following [6] the Lévy stable index μ, another measure of fractality, can be calculated from the spectral function. According to [6]: $1-f(\alpha_q) \propto (B-\alpha_q)^{\mu/(\mu-1)}$ for $\alpha_q < B$ with: $B = 1+(1-D_2)/(2^\mu-2)$, D_2 being the generalized dimension of order 2. Therefore, in the region $\alpha_q < B$, μ can be determined from the slope $C = \mu/(\mu-1)$ of the linear dependence of $\ln(1-f(\alpha_q))$ on $\ln(B-\alpha_q)$. FIG. 4 shows the results of such calculations for all three data sets. From these plots it is clear that a single straight line cannot connect all the data points, i.e., the μ value depends on the region of straight line fit. In the low q region the μ value is 3.843 ± 0.089 for experiment, as $C \approx 1$ it is undefined for UrQMD, and is -6.715 ± 0.124 for UrQMD+BEC. The corresponding values in the high q region are, respectively, 2.199 ± 0.164, 2.466 ± 0.152 and 2.613 ± 0.08. All these μ values are beyond the range of stability ($0 \geq \mu \geq 2$), as expected for a log-Lévy stable distribution.

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