$L = 0$ States of Heavy Quarkonia

Nayneshkumar Devlani* and Ajay Kumar Rai†
Department of Applied Physics, SVNIT,
Ichchhanath, Surat, Gujarat, INDIA 395 007.

Introduction

A very large amount of experimental data on the heavy quarkonium spectroscopy has been accumulated during the last decade[1]. The number of known states is constantly increasing. Some of these new states were expected from quark models but it is believed that some of these new states could be the first manifestation of the existence of exotic hadrons (tetraquarks, molecules, hybrids etc.), which are expected to exist in QCD[1–4] . In order to explore such options, a comprehensive understanding of the heavy quarkonium spectroscopy is required.

Methodology

For the quarkonia we employ the relativistic Hamiltonian in which motion of the quarks inside the meson is relativistic[5, 6]

$$H = \sqrt{\mathbf{p}^2 + m_Q^2} + \sqrt{\mathbf{p}^2 + m_{\bar{Q}}^2} + V(\mathbf{r}) \quad (1)$$

where $\mathbf{p}$ is the relative momentum of the quark-antiquark and m_Q and $m_{\bar{Q}}$ are the heavy quark masses. We expand the kinetic energy(K.E.) part of the Hamiltonian up to $\mathcal{O}(p^6)$, and $V(\mathbf{r})$ is the quark-antiquark potential [7, 8],

$$V(r) = -\frac{\alpha_c}{r} + Ar + V_0 \quad (2)$$

where A is the potential parameter and ν is a general power index, $\alpha_c = (4/3)\alpha_S$, where α_S is the strong running coupling constant. The value of α_s is determined through the simplest model with freezing, namely

$$\alpha_s(M^2) = \frac{4\pi}{\left(11 - \frac{2}{3}n_f\right)\ln\frac{M^2+M_B^2}{\Lambda^2}} \quad (3)$$

where the scale is taken as $M = 2m_Q m_{\bar{Q}}/\left(m_Q + m_{\bar{Q}}\right)$, the background mass is $M_B = 2.24\sqrt{A'} = 0.95$ GeV, $\Lambda = 413$ MeV[9].

We have used the gaussian as well as hydrogen like wave function in the present study. The gaussian wave function in position space has the form

$$R_{nl}(\mu, r) = \mu^{3/2}\left(\frac{2\,(n-1)!}{\Gamma\,(n+l+1/2)}\right)(\mu r)^l \times e^{-\mu^2 r^2/2} L_{n-1}^{l+1/2}(\mu^2 r^2) \quad (4)$$

and the hydrogen like wave function has the form

$$R_{nl}(r) = \left(\frac{\mu^3(n-l-1)!}{2n(n+l)!}\right)^{1/2} (\mu\ r)^l \times e^{-\mu r/2} L_{n-l-1}^{2l+1}(\mu r) \quad (5)$$

Here, μ is the variational parameter and L is Laguerre polynomial. Usiing the Ritz variational scheme, we obtain the expectation values of the Hamiltonian as

$$H\psi = E\psi \quad (6)$$

TABLE I: Value of V_0 (in GeV)

Charmonium		Bottomonium	
Gauss.	Hydro.	Gauss.	Hydro.
-0.287	-0.213	-0.370	-0.336

$m_c = 1.55\ GeV$, and $m_b = 4.88\ GeV$

*Electronic address: nayneshdev@gmail.com
†Electronic address: raiajayk@gmail.com

TABLE II: $L = 0$ Masses of Charmonium and Bottomonium(in GeV).

nL	Charmonium					Bottomonium				
	Our Work					Our Work				
	Gauss.	Hydro.	Expt.[1]	Ref [10]	Ref[11]	Gauss.	Hydro.	Expt[1]	Ref[10]	Ref[11]
1^1S_0	2.972	2.800	2.980	2.981	2.981	9.421	9.331	9.391	9.398	9.421
1^3S_1	3.100	3.157	3.097	3.096	3.097	9.464	9.494	9.460	9.460	9.460
2^1S_0	3.665	3.585	3.637	3.635	3.619	10.014	9.998		9.990	10.004
2^3S_1	3.692	3.695	3.686	3.685	3.686	10.023	10.033	10.023	10.023	10.023
3^1S_0	4.111	4.015		3.989	4.052	10.400	10.362		10.329	10.350
3^3S_1	4.126	4.079	4.039	4.039	4.102	10.404	10.381	10.355	10.355	10.366
4^1S_0	4.484	4.356		4.401	4.447	10.720	10.653		10.573	10.632
4^3S_1	4.494	4.401	4.421	4.427		10.723	10.666	10.579	10.586	10.643
5^1S_0	4.816	4.654		4.811		11.005	10.906		10.851	
5^3S_1	4.824	4.689		4.837		11.007	10.917	10.876	10.869	
6^1S_0	5.121	4.925		5.155		11.265	11.137		11.061	
6^3S_1	5.127	4.953		5.167		11.267	11.145	11.019	11.088	

For a chosen value of ν, the variational parameter, μ is determined for each state using the Virial theorem[12]. Eq(6) gives the spin averaged(SA) masses of the system. The spin-averaged mass is matched with the experimental value for the ground state using the equation [7]

$$M_{SA} = M_P + \frac{3}{4}(M_V - M_P) \qquad (7)$$

where M_V and M_P are the experimentally measured vector and pseudoscalar meson ground state masses. This fixes the value of V_0 which is listed in Table-(I) alongwith other parameters used. Using this value of V_0 we calculate S, wave of the $c\bar{c}$ and the $b\bar{b}$ mesons which are listed in Table (II).

Summary

We have calculated the mass spectra for $L = 0$ states of the $c\bar{c}$ and $b\bar{b}$ meson. It can be clearly seen from Table-(II) that our results for the ground state as well as excited state masses of charmonium for the gaussian wavefunction are fairly close to the experimental measurements as well as other theoretical model predictions whereas in the case of the hydrogenic wavefunction the excited states are in reasonable agreement with others. But in the case of masses for the Bottomonium meson we find that both the Gaussian as well as the hydrogenic wavefunction only agree at low lying states to the experimental measurements as well as other results. However, the agreement for excited states is found to be poor.

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