S wave $c\bar{c}$ spectrum in a semi-relativistic model

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Introduction

Quarkonium system has come into news recently with the ATLAS detector at the Large Hadron Collider (LHC) discovering the previously unobserved $\chi_b(3P)$ state [1]. Quarkonium consists of a quark and its antiquark ($Q\bar{Q}$). The study of heavy quarkonium systems has played an important role in the development of quantum chromodynamics (QCD). Heavy quarkonium decays may provide useful information on understanding the nature of inter-quark forces and decay mechanisms. Since the hadron spectrum cannot be obtained directly from QCD, one has to use other methods like potential model calculations, lattice gauge theory, effective field theory, etc., to investigate hadron spectrum and its decays. Phenomenological potential models are still one of the important tools to study the hadron spectrum and its decays. These models are either relativistic or nonrelativistic.

Model

The Hamiltonian for a free spinless relativistic two particle system is,

$$H = \sqrt{p_1^2 + m_1^2} + \sqrt{p_2^2 + m_2^2} \quad (1)$$

The corresponding relativistic wave equation can be written as

$$H\Psi = \left(\sqrt{p_1^2 + m_1^2} + \sqrt{p_2^2 + m_2^2}\right)\Psi \quad (2)$$

If Ψ is an eigenstate of H with an eigenvalue E, Eq. (2) becomes,

$$E\Psi = \left(\sqrt{p_1^2 + m_1^2} + \sqrt{p_2^2 + m_2^2}\right)\Psi \quad (3)$$

In the CM frame, the above equation becomes

$$E\Psi = 2\ \sqrt{p^2 + m^2}\ \Psi \quad (4)$$

The vector potential (V) and the scalar potential (U) are introduced in Eq. (4) through the rule: $E \to E - V$ and $m \to m + U$. We obtain,

$$(E - V)\ \Psi = 2\ \sqrt{p^2 + (m + U)^2}\ \Psi, \quad (5)$$

or,

$$(E - V)^2\ \Psi = 4\ \left(p^2 + (m + U)^2\right)\ \Psi, \quad (6)$$

With the replacement $p \to -i\nabla$ and $\Psi = R_{nl}(r)Y_{lm}(\theta, \phi) = \frac{u(r)}{r}Y_{lm}(\theta, \phi)$ and after simplification Eq. (6) reduces to [2],

$$\left[-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} + V_{eff}\right]\ u(r) = E_{eff}\ u(r) \quad (7)$$

where,

$$V_{eff} = \frac{4U^2 + 8mU + 2EV - V^2}{4} \quad (8)$$

$$E_{eff} = \frac{E^2 - 4m^2}{4} \quad (9)$$

In order to obtain the spin averaged masses of the $c\bar{c}$ system, we have solved Eq. (7) by the variational method.

The spin dependent correction used, which gives the hyperfine splitting between the vector and pseudoscalar states, is of the form [3, 4],

$$H_{SS} = \frac{2}{3m_q^2}\vec{S}_Q \cdot \vec{S}_{\bar{Q}}\ \Delta V_v(r)$$

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where m_q is the mass of the quark and Δ is the Laplacian operator. $\vec{S}_Q$ and $\vec{S}_{\bar{Q}}$ are the spin operators for the quark and antiquark respectively, with

$$\langle \vec{S}_Q \cdot \vec{S}_{\bar{Q}} \rangle = \begin{cases} \frac{1}{4} & \text{for } \vec{S} = 1 \\ -\frac{3}{4} & \text{for } \vec{S} = 0 \end{cases} .$$

TABLE I: Mass Spectrum (in MeV)

Meson	Present	Exp.[5]	[6]	[7]	[8]
J/ψ	3097	3097	3123	3096	3139
$\psi(2S)$	3683	3686	3683	3686	3694
$\psi(3S)$	4056	4039	4161	4088	4085
$\psi(4S)$	4343	4421			4412
$\psi(5S)$	4579				
$\eta_c(1S)$	2980	2980	2926	2979	3052
$\eta_c(2S)$	3617	3637	3597	3588	3655
$\eta_c(3S)$	4002		4102	3991	4057

Results and Discussion

In the present work, using a $Q\bar{Q}$ interaction, which consists of a Coulomb-like potential and a confining power-law potential, the spectra of $c\bar{c}$ mesons were computed. The results obtained agree with the experimental results and with the predictions from other theoretical models. The results are tabulated in Table I.

References

[1] The ATLAS Collaboration, arXiv: 1112.5154v4 [hep-ex].

[2] *"Charmonium spectra and decays in a semi-relativistic model"*, Bhaghyesh and K. B. Vijaya Kumar, Int. J. Mod. Phys. A, (Accepted).

[3] W. Lucha, F. F. Schöberl, arXiv: 9601263v1 [hep-ph].

[4] M. B. Voloshin, Prog. Part. Nucl. Phys., **61** 2008 455, arXiv:0711.4556v3 [hep-ph].

[5] K. Nakamura, et al., Particle Data Group, J. Phys. G: Nucl. Part. Phys. **37** (2010) 075021.

[6] A. K. Rai, B. Patel, and P. C. Vinodkumar, Phys. Rev. C **78**, (2008) 055202.

[7] D. Ebert, R. N. Faustov and V. O. Galkin Phys. Rev. D **67** (2003) 014027.

[8] O. Lakhina and E. S. Swanson, Phys. Rev. D **74** (2006) 014012.