# $\Delta$ in $\pi^0$ production in $pp$ collisions

G. Ramachandran[1],* Sujith Thomas[1,2],† and Venkataraya[1,3]‡
[1] *G. V. K. Academy, Bangalore - 560070, INDIA*
[2] *IIT Madras, Chennai-600036, INDIA and*
[3] *Vijaya College, Bangalore - 560056, INDIA*

### Introduction

Experimental and theoretical study of $pp \to pp\pi^0$ has excited considerable interest, ever since the total cross section measurements [1] were found to be more than 5 times larger than the then existing theoretical predictions. Advances in technology led to experimental studies [2, 3] employing a polarized beam on a polarized target. The Julich meson exchange model [4] was found to be more successful with the less complete data [3] on $\vec{p}\vec{p} \to d\pi^+$ and $\vec{p}\vec{p} \to pn\pi^+$, but it failed to provide an overall satisfactory reproduction of the polarization observables [2] in the case of $\vec{p}\vec{p} \to pp\pi^0$.

When the Julich meson exchange model was confronted [5] with a model independent analysis [6] of the data [2], it led to an identification of partial waves where the model is deficient. Moreover, it revealed that $\Delta$ contributions are important. Because the final spin singlet and triplet states do not mix in any of these measured [2] spin observables, both the $Ss$ and $Ps$ partial wave amplitudes were assumed in [5] to be real. It was shown [7] how this drawback could be removed, if final state spin observables are measured.

A model independent approach to discuss $\Delta$ contributions was formulated more recently [8], where the irreducible tensor amplitudes were expressed in terms of nine partial wave amplitudes. These amplitudes are not of the same partial wave amplitudes employed in [2, 7].

The purpose of the present paper is to show how the reaction amplitudes used in [2, 7] and those defined in [8] may be related to each other.

### Theory

When $\Delta$ with mass $M_\Delta$ is produced, the cm energy $E$ for the reaction gets divided into the cm energies

$$E_\Delta = \frac{1}{2E}(E^2 - M^2 + M_\Delta^2) \qquad (1)$$

$$E_N = \frac{1}{2E}(E^2 + M^2 - M_\Delta^2) \qquad (2)$$

of the $\Delta$ and a proton in the final state. The masses of the proton and the meson are denoted by $M$ and $m$ respectively. Let $\mathbf{p}_1, \mathbf{p}_2, \mathbf{q}$ denote the cm momenta of the two nucleons and the pion in the final state and $E_1, E_2, \omega$ denote their respective cm energies. To discuss $\Delta$ contributions, one may choose events where (say) $E_2 = E_N$, so that $\mathbf{p}_1 + \mathbf{q} = \mathbf{p}_\Delta$.

The reaction amplitudes employed in [2, 7] and those defined in [8] start from the same initial state $|(l_i s_i)jm>$, but the description of the final state is based on two different ways of defining the Jacobi coordinates. If $\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2$ denote the instantaneous locations of the meson and the two nucleons, the Jacobi co-ordinates

$$\mathbf{R}_1 = \frac{1}{M+m}(m\mathbf{r} + M\mathbf{r}_1) \qquad (3)$$

*Electronic address: gwrvrm@yahoo.com
†Electronic address: mailsujiththomas@gmail.com
‡Electronic address: venkataraya@gmail.com

$$\mathbf{r'}_1 = \mathbf{r} - \mathbf{r}_1; \quad \mathbf{r'}_2 = \mathbf{r}_2 - \mathbf{R}_1 \tag{4}$$

are used in [8], so that the momentum $\mathbf{q}'$ of the $P$ wave pion in $\Delta$ is given by

$$\mathbf{q}' = \frac{Mm}{M+m}\frac{d\mathbf{r'}_1}{dt} = \frac{M}{M+m}\mathbf{q} - \frac{m}{M+m}\mathbf{p}_1 \tag{5}$$

while, the relative momentum between the nucleon and the $\Delta$ is given by

$$\frac{(M+m)M}{2M+m}\frac{d\mathbf{r'}_2}{dt} = \mathbf{p}_2. \tag{6}$$

The total orbital angular momentum and total spin in the scheme of [8] are given by

$$\mathbf{L} = \mathbf{r'}_1 \times \mathbf{q}' + \mathbf{r'}_2 \times \mathbf{p}_2; \; \mathbf{S} = \mathbf{S}_\Delta + \mathbf{S}_2 \tag{7}$$

The spin $s_\Delta = \frac{3}{2}$ and $s_2 = \frac{1}{2}$ combine together to give $s = 1, 2$. The partial wave amplitude employed in [8] may explicitly be written as $M^j_{l_2((l_1 s_1)s_\Delta s_2)s; l_i s_i}$ if $l_1, l_2$ are the quantum numbers associated with the first and second terms of $\mathbf{L}$ in Eq.(7). The angular dependance of the irreducible tensor amplitudes in [8] may therefore be reduced to the form

$$\left((Y_{l_1=1}(\hat{\mathbf{q}'}) \otimes Y_{l_2}(\hat{\mathbf{p}}_2))^{L_f} \otimes Y_{l_i}(\hat{\mathbf{p}}_i)\right)^\lambda_\mu \tag{8}$$

where, we may express $Y_{1m}(\hat{\mathbf{q}'})$ as

$$Y_{1m}(\hat{\mathbf{q}'}) = \frac{1}{(M+m)|\mathbf{q}'|} \times [M|\mathbf{q}|Y_{1m}(\hat{\mathbf{q}}) - m|\mathbf{p}_1|Y_{1m}(\hat{\mathbf{p}}_1)] \tag{9}$$

using Eq.(5).

On the other hand, the angular dependance [7] associated with the irreducible tensor amplitudes is of the form

$$\left((Y_{l_f}(\hat{\mathbf{p}}_f) \otimes Y_l(\hat{\mathbf{q}}))^{L_f} \otimes Y_{l_i}(\hat{\mathbf{p}}_i)\right)^\lambda_\mu \tag{10}$$

where $\mathbf{p}_f = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2)$.
In the list of partial waves given in table 1 of [7] $l_f$ is limited to $0, 1$ and we may express

$$Y_{1m}(\mathbf{p}_f) = \frac{1}{2\,|\mathbf{p}_f|}\left[|\mathbf{p}_1|\, Y_{1m}(\hat{\mathbf{p}}_1) - |\mathbf{p}_2|\, Y_{1m}(\hat{\mathbf{p}}_2)\right] \tag{11}$$

Thus the irreducible tensor amplitudes in [7] as well as in [8] are expressible in terms of spherical harmonics with arguments $\hat{\mathbf{p}}_1, \hat{\mathbf{p}}_2$ and $\hat{\mathbf{q}}$ and thus brought into the form where it is possible to relate the two sets of partial wave amplitudes with each other. Details will be presented.


[1] H.O.Meyer *et al.*, Phys. Rev. Lett. **6**5, 2846 (1990);Nucl. Phys.**A539**, 633 (1992); A.Bondar *et al.*, Phys. Lett.B **356**, 8 (1995)

[2] H.O.Meyer *et al.*, Phys. Rev. C **63**, 064002 (2001)

[3] V.von Prezowski *et al.*, Phys. Rev. C **61**, 064604 (2000); W.W.Daehnick *et al.*, Phys. Rev. C **64**, 024003 (2002)

[4] C.Hanhart, J.Haidenbauer, O.Krehl and J.Speth, Phys. Lett. B**444**, 25 (1998); C.Hanhart, J.Haidenbauer, O.Krehl and J.Speth, Phys. Rev. C**61**, 064008 (2000);

[5] P.N.Deepak, J.Haidenbauer and C.Hanhart, Phys. Rev. C **72**, 024004 (2005)

[6] G.Ramachandran, P.N.Deepak and M.S.Vidya, Phys. Rev. C **62**, 011001(R) (2000); G.Ramachandran and P.N.Deepak, Phys. Rev. C **63**, 051001(R) (2001); P.N.Deepak and G.Ramachandran, Phys. Rev. C **63**, 027601 (2002); P.N.Deepak, G.Ramachandran and C.Hanhart, Matter. Mater. **21**, 138 (2004); P.N.Deepak, C.Hanhart, G.Ramachandran and M.S.Vidya, Int. J. Mod. Phys. A**20**, 599 (2005)

[7] G Ramachandran, G Padmanabha and Sujith Thomas, Proceedings of International Symposium on Nuclear Physics **54**, 474 (2009);Phys. Rev. C **81**, 067601 (2010)

[8] G Ramachandran, Venkataraya and Sujith Thomas, Proceedings of the DAE Symposium on Nuclear Physics **56**, 860 (2011)