P−wave and D−wave masses of Bottonium in a phenomenological confinement Scheme

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Introduction

Precise experimental observations of various hadronic states [1] of quarkonia have generated interests to explore the predictability of phenomenological confinement schemes. Particularly, the recent discovery of the η_b $(1S)$, $h_b(1P)$, 1^3D_2, $h_b(2P)$, $h_b(3P)$ states of bottonia [2, 3] provide a test for the success of any phenomenological scheme describing the $Q\bar{Q}$ bound states. It also provides the exact nature of hyperfine, spin-orbit and tensor interactions in the case of $b\bar{b}$ system. In this context, we consider the bottonium states and study the S-wave, P-wave, D-wave mass spectra.

Methodology

It has been shown that a purely phenomenological approach to the nonrelativistic potential-model study of Υ spectra can lead to a static non-Coulombic Power-law potential of the form [4, 5]

$$V(r) = \lambda r^{\nu} + V_{\circ} \tag{1}$$

where ν is chosen to be 0.1 for Martin like potential with $\lambda > 0$.

Following general quantum mechanical rules applicable to power like potentials as discussed in [6], the binding energy of a system is expressed as

$$E_{nl} = \lambda^{2/(2+\nu)} \quad (2\mu)^{-\nu/(2+\nu)} \left[A(\nu)\left(n+\frac{l}{2}-\frac{1}{4}\right)\right]^{2\nu/(2+\nu)} \tag{2}$$

and the corresponding square of the probability amplitude of the S-waves at the zero separation of the quark-antiquark system is given by

$$|\psi_{nl}(0)|^2 = \frac{1}{2\pi^2}\left(\frac{2\mu\lambda}{\hbar^2}\right)^{\frac{3}{(2+\nu)}} \frac{\nu}{(2+\nu)}[A(\nu)]^{\frac{3\nu}{(2+\nu)}} \left(n+\frac{l}{2}-\frac{1}{4}\right)^{2(\nu-1)/(2+\nu)} \tag{3}$$

where

$$A(\nu) = \left[2\nu\sqrt{\pi}\ \Gamma\left(\frac{3}{2}+\frac{1}{\nu}\right)\right]/\Gamma(1/\nu), \quad \nu > 0. \tag{4}$$

For P, D, ... waves, the radial wave function $R_{nl}(r)$ as $r \to 0$ behaves as [6]

$$R_{nl}(r) \sim a_{nl}\ r^l \tag{5}$$

So, the l^{th} derivatives of the radial wave function $R_{nl}(r)$ at zero separation of the quark-antiquark system goes as

$$R^{(l)}_{nl}(0) = l!\ a_{nl} \tag{6}$$

where

$$a_{nl} = \left[\frac{(2\mu\ E_{nl})^{l+\frac{1}{2}}}{\pi((2l+1)!!)^2}\ \frac{4\mu\nu}{(2+\nu)}\ \lambda^{\frac{2}{2+\nu}} (2\mu)^{\frac{-\nu}{2+\nu}}\ [A(\nu)]^{\frac{2\nu}{2+\nu}}\left(n+\frac{l}{2}\right)^{\frac{\nu-2}{\nu+2}}\right]^{\frac{1}{2}} \tag{7}$$

Now the nonrelativistic Schrodinger bound-state mass (spin average mass) of the $Q\bar{Q}$ $(Q \in b, c)$ system is expressed as

$$M_{SA} = 2m_Q + V_0 + E_{nl} \tag{8}$$

For the hyperfine split we have considered the standard one gluon exchange interactions

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TABLE I: Mass spectra of $b\bar{b}$ spectrum (in MeV)

nS	[our]	[11]	[7]	Exp. [1]
1^3S_1	9460.43	9460.38	9472	$\Upsilon(9460.30)$
1^1S_0	9392.38	9392.91	9399	$\eta_b(9390.7)$ [12]
2^3S_1	10023.80	10023.3	9951	$\Upsilon(10023.26\ )$
2^1S_0	9990.88	9987.42	9924	-
3^3S_1	10345.80	10364.2	10334	$\Upsilon(10355.2)$
3^1S_0	10323.40	10333.9	10317	-
4^3S_1	10575.50	10636.4	-	$\Upsilon(10579.4)$
4^1S_0	10558.30	10609.4	-	-
1^3P_2	9907.89	9910.63	9866	$\chi_{b2}(9912.21)$
1^3P_1	9887.63	9891.33	9842	$\chi_{b1}(9892.78)$
1^3P_0	9862.29	9861.39	9820	$\chi_{b0}(9859.44)$
1^1P_1	9896.07	9899.93	9852	$h_b(9900.00)$ [2]
2^3P_2	10267.65	10271.2	10282	$\chi_{b2}(10268.65)$
2^3P_1	10255.74	10254.8	10246	$\chi_{b1}(10255.46)$
2^3P_0	10240.85	10230.5	10228	$\chi_{b0}(10232.50)$
2^1P_1	10260.70	10261.8	10264	h_b (10259.8) [14]
3^3P_2	10516.28	-	-	-
3^3P_1	10507.74	-	-	-
3^1P_1	10497.07	-	-	-
3^3P_0	10511.30	-	-	$h_b(10551)$ [3]

[7–9]. Accordingly, the hyperfine mass split for the S-wave is given by

$$\Delta M = A_{hyp}|\psi_n(0)|^2/m_Q^2. \qquad (9)$$

Further, we have considered the tensor and spin-orbit components of the Breit interaction terms as given by [8–10]. The present results obtained for higher S− wave, P− wave and D− wave masses are listed in Table I.

Results and discussion

We have computed bottonium spectra which are in very good agreement with the reported PDG values of known states. Present results are also found to be in good agreement with other theoretical predictions based on semi-relativistic method [11]. The only bottonia D-state known experimentally is 1^3D_2 $(10163.8 \pm 1.4 MeV)$ reported by BABAR collaboration [15] which is in close agreement with our predicted value of 10162 MeV. For better comparison with the experiment, the energy levels of bottonia states are shown in FIG.(1).

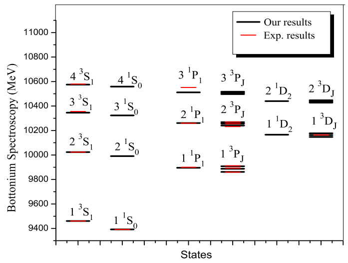

FIG. 1: Bottonium Spectroscopy

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