Ground State Properties of The B_c Meson

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Introduction

Experimental measurements of the masses of the B_c meson beyond the ground state pseudoscalar are scarce[1]; however new experimental data on the spectroscopy of B_c meson is expected from LHCb[2]. The B_c meson, within the quark model is considered to be a heavy-heavy quark bound system. But the c quark is lighter in mass in comparison with the b quark. In order to understand the properties of this meson the presence of the light quark necessitates relativistic treatment. In the present paper in order to study the ground state properties of the B_c meson, we use a semi-relativistic treatment.

Methodology

For the B_c meson we employ the relativistic Hamiltonian in which motion of the quarks inside the meson is relativistic[3, 4]

$$H = \sqrt{\mathbf{p}^2 + m_Q^2} + \sqrt{\mathbf{p}^2 + m_{\bar{q}}^2} + V(\mathbf{r}) \quad (1)$$

where $\mathbf{p}$ is the relative momentum of the quark-antiquark and m_Q is the heavy quark mass and $m_{\bar{q}}$ is the light quark mass. We expand the kinetic energy(K.E.) part of the Hamiltonian up to $\mathcal{O}(p^6)$, and $V(\mathbf{r})$ is the quark-antiquark potential [5–7],

$$V(r) = -\frac{\alpha_c}{r} + Ar + V_0 \quad (2)$$

where A is the potential parameter, $\alpha_c = (4/3)\alpha_S$, where α_S is the strong running coupling constant. The value of α_s is determined through the simplest model with freezing, namely

$$\alpha_s(M^2) = \frac{4\pi}{\left(11 - \frac{2}{3}n_f\right)\ln\frac{M^2+M_B^2}{\Lambda^2}} \quad (3)$$

where the scale is taken as $M = 2m_Q m_{\bar{q}}/\left(m_Q + m_{\bar{q}}\right)$, $M_B = 0.95$ GeV, $\Lambda = 413$ MeV[8].

We have used the gaussian as well as hydrogen like wave function in the present study.The gaussian wave function in position space has the form

$$R_{nl}(\mu, r) = \mu^{3/2}\left(\frac{2\,(n-1)!}{\Gamma\,(n+l+1/2)}\right)(\mu r)^l \times e^{-\mu^2 r^2/2} L_{n-1}^{l+1/2}(\mu^2 r^2) \quad (4)$$

and the hydrogen like wave function has the form

$$R_{nl}(r) = \left(\frac{\mu^3 (n-l-1)!}{2n(n+l)!}\right)^{1/2} (\mu\, r)^l \times e^{-\mu r/2} L_{n-l-1}^{2l+1}(\mu r) \quad (5)$$

Here, μ is the variational parameter and L is Laguerre polynomial. Usiing the Ritz variational scheme, we obtain the expectation values of the Hamiltonian as

$$H\psi = E\psi \quad (6)$$

The variational parameter, μ is determined for each state using the Virial theorem[9]. Eq(6) gives the spin averaged(SA) mass of the system. The spin-averaged mass is matched with that of the Ref [8] for the ground state using the equation [7]

$$M_{SA} = M_P + \frac{3}{4}(M_V - M_P) \quad (7)$$

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TABLE I: Ground state masses of the B_c meson.

State	V_0(in GeV)		$\|R(0)\|$		$Mass$(in Gev)		Expt.[1]	Ebert[10]
	Gauss.	Hydro.	Gauss	Hydro.	Gauss.	Hydro.		
1^1S_0	-0.204	-0.171	1.087	2.095	6.271	6.144	6.277	6.272
1^3S_1	-0.204	-0.171	1.098	2.179	6.334	6.376		6.333

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TABLE II: Decay constants of the B_c meson(in GeV).

f_P		f_V	
Gauss	Hydro	Gauss	Hydro
0.424	0.826	0.426	0.843
(0.339)	(0.660)	(0.340)	(0.340)
0.433[12]		0.503[12]	
		0.418 ± 0.024[13]	

where M_V and M_P are the experimentally measured vector and pseudoscalar meson ground state masses. This fixes the parameter V_0. Using this value of V_0 we calculate S, wave masses of B_c meson which are listed in table I. Masses used for the calculation are $m_b = 4.88\ GeV$, $m_{\bar{c}} = 1.55\ GeV$.

Decay Constants

Incorporating a First order QCD correction factor , we compute the decay constants using the relation,

$$f^2_{P/V} = \frac{12\left|\psi_{P/V}(0)\right|^2}{M_{P/V}}\bar{C}^2(\alpha_S) \qquad (8)$$

Where $\bar{C}^2(\alpha_S)$ is the QCD correction factor given by[11]

$$\bar{C}^2(\alpha_S) = 1 - \frac{\alpha_S}{\pi}\left[2 - \frac{m_Q - m_{\bar{q}}}{m_Q + m_{\bar{q}}}\ln\frac{m_Q}{m_{\bar{q}}}\right] \quad (9)$$

Calculated decay constants are listed in table-(II) Bracketed values are the decay constants with QCD correction.

Summary

Looking at the results from table-(I) we find that our results for the B_c meson ground state masses for the Gaussian wave function are in excellent agreement with the experimental results as well as results of ref [10]. While looking at table-(II) we find that again the decay constants calculated using the Gaussian wave function without the QCD correction are in good agreement with others.

However in the case of hydrogenic wave function we find that due to the large value of the wave function at the origin both the mass spectra and decay constants are not in good agreement with experimental results. Thus, it is found that the Gaussian wave function is more suitable for the study of the B_c meson within the current framework.

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