Optical Activity From High Energy Physics Models

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I. INTRODUCTION

Since the last decade we have come across some observational evidence [1, 2] suggest that the universe is currently undergoing accelration. A way to resolve this problem is by introducing a scalar field that provides "dark energy" with negative pressure, that couples to ordinary matter fields. There are many theories where the existense of light scalar fields is possible, e.g. in string theory there are many moduli fields that couple to matter or scalar tensor theory etc. One such theory goes by the name of the chameleonic theory. The introduction of chemeleon field was to explain to the source of dark matter in the universe. Interaction of chameleon with electromagnetic field was suggested in [4–6]. The chameleon photon action in a strong magnetic field is given by:

$$\begin{aligned} S &= \int d^4x\sqrt{-g}\Big[\frac{1}{2}M_{\rho l}^2 R + \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi) \\ &- \frac{1}{4}e^{\frac{\beta_\gamma\phi}{M_{\rho l}}}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha^2}{90M_e^4}[(F_{\mu\nu}F^{\mu\nu})^2 \\ &+ \frac{7}{4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2\Big]. \end{aligned} \quad (1)$$

Here β_γ is the effective coupling strength between chameleon field and photon, V(ϕ) is the effective potential. For the chameleonic field for which one purpose would be taken as $\frac{1}{2}M^2\phi^2$. Here M is the mass of the chameleon field. Though in general, mass of chameleon depends on the density of the ambient medium, however we are considering a situation where the density of the medium is uniform and hence mass of the chameleon can be taken to be constant. For our purpose we are working in flat space and we would retain terms up to $\frac{1}{M_{\rho l}}$ in expansion of the exponential $e^{\frac{\beta_\gamma\phi}{M_{\rho l}}}$. In order to make things look simpler, we can intoduce a short notation to denote $\frac{\alpha^2}{90M_e^4} = \alpha\prime^2$. We are interested in a situation where there is strong magnetic field, however the gravitational effects are not so important. There are many astrophysical objects where the strength of the background magnetic field can lie between $10^9 - 10^{15}$ Gauss. Therefore, the study of the dispersion relation of photon in such an environment is likely to bear the signature of Chameleon field, if it is present. The analysis of the dispersion relation assuming a simplest kind of photon chameleon interaction shows the presence of two modes, one of which is subluminal and the other superluminal. As already stated before, the physical situation considered in this analysis is relevant for astrophysical environment and might have some interesting consequences if combined with observations.

II. EQUATION OF MOTION

Let us assume that the space-time to be flat, which is a justified assumption if we are interested in the emission zone of the steller objects which are for from the surface. With this assumption one can take $\sqrt{-g} = 1$. The metric tensor under consideration is $g_{\mu\nu} = diag(1, -1, -1, -1)$. On appling variational principle, the equation of motion for photon field is:

$$\partial_\mu[F^{\mu\nu} - 8\alpha\prime^2((F.F)F^{\mu\nu} + \frac{7}{4}(F.\tilde{F})\tilde{F}^{\mu\nu}) + g_{c\gamma\gamma}\phi F^{\mu\nu}] = 0.(2)$$

The second equation of motion for chameleon field, by the variation of ϕ is:

$$\partial_\mu\partial^\mu\phi - m^2\phi - \frac{1}{4}\,g_{c\gamma\gamma}F_{\mu\nu}F^{\mu\nu} = 0. \quad (3)$$

We would further assume that the electromagnetic field can be decomposed into two parts i.e. $F^{\mu\nu} = \bar{F}^{\mu\nu} + f^{\mu\nu}$. The first part $\bar{F}^{\mu\nu}$ is mean field and the second part $f^{\mu\nu}$ is the fluctuation. Substituting the value of $F^{\mu\nu} = \bar{F}^{\mu\nu} + f^{\mu\nu}$ in above equation and put the value of $\frac{\beta_\gamma}{M_{\rho l}} = g_{c\gamma\gamma}$, where β_γ is dimensionless coupling constants and $M_{\rho l}$ is Planck mass. Here $g_{c\gamma\gamma}$ is chameleon-electomagnetic field coupling constant. The equation of motion for photon field is written as:

$$\begin{aligned} \triangle\partial_\mu f^{\mu\nu} - 16\alpha\prime^2\partial_\mu(\bar{F}_{\lambda\rho}f^{\lambda\rho})\bar{F}^{\mu\nu} - 28\alpha\prime^2\partial_\mu(\tilde{\bar{F}}_{\lambda\rho}f^{\lambda\rho})\tilde{\bar{F}}^{\mu\nu} \\ = -g_{c\gamma\gamma}\partial_\mu\phi\bar{F}^{\mu\nu}.(4) \end{aligned}$$

Where we have use the following shorthand notation, $\triangle = [1 - 8{\alpha'}^2(\bar{F}_{\lambda\rho}\bar{F}^{\lambda\rho})]$.

Let $\psi = \frac{f^{\lambda\rho}}{2}\bar{F_{\lambda\rho}}$. Put this term in the equation (4), we get a new equation in terms of $\psi = f^{\nu\lambda}\bar{F_{\nu\lambda}}$ field. Now using the Bianchi identity and solving the above equation. Therefore, the new equation of motion is written as

$$\begin{aligned} \triangle\partial^\mu\partial_\mu\psi + 32\alpha\prime^2\partial^\lambda\partial_\mu\bar{F}_{\nu\lambda}\bar{F}^{\mu\nu}\partial_\mu\psi \\ = g_{c\gamma\gamma}(\partial^\lambda\partial_\mu\phi)(\bar{F^{\mu\nu}}\bar{F_{\nu\lambda}}), \end{aligned} \quad (5)$$

$$\partial_\mu\partial^\mu\phi + m^2\phi + g_{c\gamma\gamma}\psi = 0. \quad (6)$$

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We can find another equation of motion if we multiply by dual tensor $\tilde{\tilde{F}}^{\nu\lambda}$ in the equation of motion and taking partial differentiation with respect to lambda i.e. ∂_λ. Here the expression $\bar{F}_{\mu\nu}\tilde{\bar{F}}^{\nu\lambda} = 0$. Since $\tilde{F}_{\nu\lambda} = \frac{1}{2}\epsilon_{\alpha\beta\nu\lambda}F^{\alpha\beta}$, where $\epsilon_{\alpha\beta\nu\lambda}$ is Levi-civita tensor.

$$\triangle\partial_\mu\partial^\mu\tilde{\psi} + 28\alpha\prime^2\partial_\lambda\partial^\mu(\tilde{\bar{F}}_{\mu\nu}\tilde{\bar{F}}^{\nu\lambda})\tilde{\psi} = 0 \tag{7}$$

III. DISPERSION RELATION

By using Fourier transformation, the equations of motion for the photon and chameleon degree of freedom can be written in terms of momentum space:

$$k^2\tilde{\psi} + 28\alpha'^2(\omega^2 Sin^2\Theta)B^2\tilde{\psi} = 0, \tag{8}$$

$$k^2\psi + 32\alpha'^2(\omega^2 Sin^2\Theta)B^2\psi = g_{c\gamma\gamma}(\omega^2 Sin^2\Theta)B^2\phi, \tag{9}$$

$$(k^2 - m^2)\phi = g_{c\gamma\gamma}\psi. \tag{10}$$

All the three equations are written in the matrix form

$$\begin{bmatrix} k^2 + \frac{7}{2}\xi(2\omega^2 Sin^2\Theta) & 0 & 0 \\ 0 & k^2 + \frac{4}{2}\xi(2\omega^2 Sin^2\Theta) & -g_{c\gamma\gamma}(\omega^2 Sin^2\Theta)\mathcal{B} \\ 0 & -g_{c\gamma\gamma} & k^2 - m^2 \end{bmatrix} \begin{bmatrix} \tilde{\psi} \\ \psi \\ \phi \end{bmatrix} = 0. \tag{11}$$

IV. RESULT

Various optical parameters like polarization, ellipcity and degree of polarization of a given light beam can be found from the the coherencey matrix constructed from various correlation functions given above. The coherency matrix, for a system with two degree of freedom is defiend as an ensamble average of direct product of two vectors

$$\begin{aligned} \rho(z) &= \langle \begin{pmatrix} A_{||}(z) \\ A_\perp(z) \end{pmatrix} \otimes \left(A_{||}(z)\ \ A_\perp(z)\right)^* \rangle \\ &= \begin{pmatrix} \langle A_{||}(z)A^*_{||}(z)\rangle & \langle A_{||}(z)A^*_\perp(z)\rangle \\ \langle A^*_{||}(z)A_\perp(z)\rangle & \langle A_\perp(z)A^*_\perp(z)\rangle. \end{pmatrix} \end{aligned} \tag{12}$$

The stokes parameters:

$$\begin{aligned} \mathrm{I} &= < A^*_{||}(z)A_{||}(z) > + < A^*_\perp(z)A_\perp(z) >, \\ \mathrm{Q} &= < A^*_{||}(z)A_{||}(z) > - < A^*_\perp(z)A_\perp(z) >, \\ \mathrm{U} &= 2\mathrm{Re} < \mathrm{A}^*_{||}(\mathrm{z})\mathrm{A}_\perp(\mathrm{z}) >, \\ \mathrm{V} &= 2\,\mathrm{Im} < \mathrm{A}^*_{||}(\mathrm{z})\mathrm{A}_\perp(\mathrm{z}) > . \end{aligned} \tag{13}$$

The ellipticity angle, χ, following can be shown to be equal to,

$$\tan 2\chi = \frac{\mathrm{V}}{\sqrt{\mathrm{Q}^2 + \mathrm{U}^2}}, \tag{14}$$

and the polarization angle can be shown to be equal to

$$\tan 2\psi\prime = \frac{\mathrm{U}}{\mathrm{Q}}. \tag{15}$$

[1] E.Komatsu et.al.Ap. J. Suppl. **180**, 330 (2009).
[2] J.Dunkley et.al.,Ap. J. Suppl. **180**, 306 (2009).
[3] P.Brax,C.van de Bruck,A.CDavis,J.khoury and A.Weltman, Phys.Rev.D70,123518(2004).
[4] J.khoury and A.Weltman phys.Rev.Lett.93,171104(2004),
[5] J.Khoury and A.Weltman phys.Rev.D69, 044026(2004).
[6] P.Brax,C.Van de Bruck and A.C Davis,Phys.Rev.Lett.99,121103(2007),A.C Davis C.A.O.Schelpe and D.j.Shaw,[arXiv/0907.2672] and references there in.
[7] J.Schwinger,Phys.Rev82, 664(1951).