

## Microscopic shell model study of Na chain

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### Introduction

The ab initio methods have been proven very successful in describing the nuclear structure properties for low mass or  $p$ -shell nuclei starting from the realistic or microscopic interactions [1]. Because of computational limitations, the properties of the medium and higher mass nuclei can not be studied using these methods. Alternatively, the nuclear structure properties of such nuclei can be studied in the shell model approach using effective interactions derived for a particular model space from the microscopic interactions in the ab initio approaches [2, 3].

From the shell model configuration, the valance nucleons of Na ( $Z=11$ ) lie in  $sd$ -shell. It has a single or doubly open shell depending on the number of neutrons (even or odd). The neutron-rich Na isotopes also lie in the “island of inversion” region. The structural properties exhibited by such nuclei are different from those with closed shell configurations and should be discussed from microscopic effective interactions. We have comprehensively studied ground state (g.s) energies, relative low-lying energy spectra, electric reduced transition strengths, quadrupole, and magnetic moments of Na chain using various microscopic effective interactions derived for a particular model space. The microscopic effective interactions N3LO and Valance Space (VS) IM-SRG are used to study Na isotopes. Additionally, we have also used the phenomenological USDB interactions and compared the results of these effective interactions with experimental data [5].

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### Formalism

The microscopic effective interaction N3LO is derived using NCSM formalism, while the VS IM-SRG interaction from the IM-SRG approach.

The A-body Hamiltonian in NCSM formalism is given by

$$H_A = \frac{1}{A} \sum_{i < j}^A \frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i < j}^A V_{ij}^{NN} \quad (1)$$

where the first term represents the relative nucleon-nucleon kinetic energy and the second term is the  $NN$  potential. The effective interaction N3LO, projected for a particular model space, is obtained by applying two times OLS (Okubo-Lee-Suzuki) transformation on the original Hamiltonian [2].

In the IM-SRG approach, the Similarity Renormalization Group (SRG) flow equation (Eq.2) is used to decouple the effective interaction for a particular model space from the original Hamiltonian containing  $NN$  and  $NNN$  forces. Here ‘s’ is known as the flow parameter, and  $\eta(s)$  is called the anti-hermitian generator. The VS-interaction can be obtained by choosing a suitable form of  $\eta(s)$  and solving the flow equation [4].

$$\frac{dH(s)}{ds} = [\eta(s), H(s)]. \quad (2)$$

Then these effective interactions are used in the shell model approach to study the nuclear structure properties of the Na chain.

### Results and Discussion

The g.s energies obtained from N3LO, IM-SRG, and USDB interactions are compared with the experimental data. The g.s energies calculated from USDB interaction are in excellent agreement with the observed value. The microscopic effective interactions also well

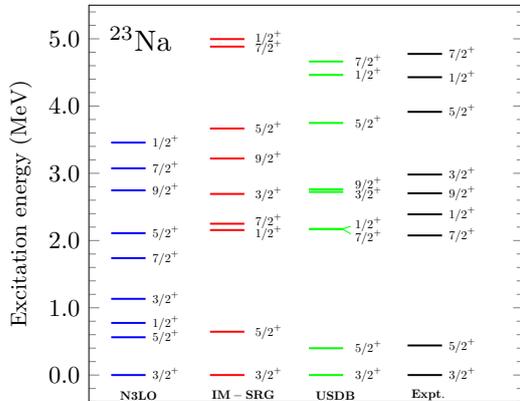


FIG. 1: Comparison between calculated and experimental energy levels for  $^{23}\text{Na}$ .

reproduce the g.s energies. However, for neutron-rich Na isotopes, both N3LO and IM-SRG interaction overbinds the g.s energy. But the overbinding is less with IM-SRG interaction compared to N3LO. The low-energy spectra of the Na isotopes are calculated from these effective interactions. The spin-parities of the g.s and the first few excited states agree with the experimental data. As an example, we have shown the energy spectra of  $^{23}\text{Na}$  in Fig. 1 and present a discussion for it. All the effective interactions correctly reproduce the spin-parities of the g.s and the first excited state. The N3LO interaction fails to produce the ordering of higher energy levels as per the experimental spectra. In the energy spectra of USDB interaction, the  $7/2_1^+$  and  $1/2_1^+$  states are in the same order as observed in the experiment, but the energy gap between them is less than the experimental value. Though the  $7/2_1^+$  and  $1/2_1^+$  states are found inverted with IM-SRG spectra, it can still explain the relative energy gap appearing after the first excited state.

To study the collective properties of Na isotopes, we have calculated the reduced E2 strengths for selected transitions for Na isotopes using the effective interactions and compared them with the available experimental data. For example in  $^{23}\text{Na}$ , the  $B(E2; 7/2_1^+ \rightarrow$

$3/2_1^+)$  value obtained from USDB interaction is  $47.4 e^2 fm^4$  which well agrees with the experimental value  $46.7 e^2 fm^4$ . The E2 strength for this transition calculated from IM-SRG interaction is  $49.5 e^2 fm^4$ , which is also close to the observed value, but N3LO interaction predicts a larger value of  $57.7 e^2 fm^4$  for this transition.

In addition, we have also calculated the g.s quadrupole (Q) and magnetic moments ( $\mu$ ) for the Na chain from the effective interactions and compared them with experimental data. The observed g.s magnetic moment of  $^{23}\text{Na}$  is  $2.217 \mu_N$ . We obtained a slightly higher value of  $2.297 \mu_N$  from N3LO interaction for this state, while both IM-SRG and USDB interaction predicts smaller values than the experimental data  $2.052 \mu_N$  and  $2.098 \mu_N$ , respectively. The g.s quadrupole moment of  $^{23}\text{Na}$  calculated from the USDB interaction is  $0.118 eb$  slightly higher than the experimental value  $0.104 eb$ . At the same time, we obtained enhanced values of quadrupole moments  $0.135 eb$  and  $0.123 eb$  with N3LO and IM-SRG interaction, respectively. In the case of neutron-rich nuclei, where the experimental data are not available, we have reported our calculated results. The detailed calculated results for the Na chain will be presented at the conference.

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