Energy and angular distribution of cosmic muons

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Introduction

The cosmic ray flux at the top of the earth atmosphere is about 1000 particles/m² sec; predominantly consisting of protons, alpha particles and heavier nuclei known as primary cosmic radiation. They interact with air molecules (mainly oxygen and nitrogen nuclei) at the top of the atmosphere and produce secondary particles most abundantly pions and also kaons, hyperons, charmed particles and nucleon-antinucleon pairs. Some of the charged pions decay via weak force into muon and neutrino as shown below; at higher energies contribution from kaon decays also add up to the observed muon flux.

\[ \pi^- \rightarrow \mu^- + \bar{\nu}_\mu, \quad \pi^+ \rightarrow \mu^+ + \nu_\mu, \]

Study of cosmic ray muon flux is of paramount importance. They provide knowledge about primary cosmic ray flux distribution. In recent days, for rare event high energy experiments like neutrino and Dark matter one needs to have fair knowledge about cosmic ray muon flux. Cosmic ray muon flux has an energy as well as angular (zenith angle) dependence. In this article, we propose a fit function with parameters having physical significance.

The distribution of flux

Here we propose a fit function of the form

\[ \phi(E, \theta) = I_0 N (E_0 + E)^{-n} \left( 1 + \frac{E}{E_1} \right)^{-1} D^{-n+1} \]

Where \( I_0 \) is the yield of integrated flux over energy and \( N = (n - 1)E_0^{(n-1)} \). \( E_0 \) gives the total energy loss and parameter \( E_1 \) is the energy above which pion and kaon do not have contribution to the muons at sea level. The energy integrated flux at zenith angle \( \theta \) is obtained as

\[ \phi(\theta) = \phi_0 D^{-n+1} \]

\( D \), the ratio of inclined to vertical distance;

\[ D = \sqrt{R^2 \cos^2 \theta + 2R + 1} - R \cos \theta \]

Where \( R \) is the ratio of radius of earth to vertical distance.

Discussion and Conclusion

We fit the experimental data for cosmic ray muon and proton with the proposed fit function keeping all the parameters free. Fig. 1 to 3 show the muon flux distributions as a function of momentum at 0° at sea level [1, 2], at 75° at sea level [3] and at 0° at 600m altitude [4], respectively. Fig 4 shows proton flux at top of the atm as a function of momentum [5] and Fig 5 shows muon flux distribution as a function of zenith angle at sea level [6]. The best fit parameter values are listed in Table-I and II.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \mu ) at 0° sea level</th>
<th>( \mu ) at 0° at 600m</th>
<th>( \mu ) at 75° sea level</th>
<th>( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_0 ) (( m^{-2} )( s^{-1} )( sr^{-1} ))</td>
<td>88.5</td>
<td>109</td>
<td>80.99</td>
<td>1.03 x 10⁴</td>
</tr>
<tr>
<td>( n )</td>
<td>3.00</td>
<td>3.09</td>
<td>3.07</td>
<td>2.88</td>
</tr>
<tr>
<td>( E_0 (GeV) )</td>
<td>4.2</td>
<td>4.03</td>
<td>24.5</td>
<td>1.18</td>
</tr>
<tr>
<td>( \frac{E_0}{E} (GeV^{-1}) )</td>
<td>1/736</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>( R )</td>
<td>488</td>
<td>1</td>
<td>488</td>
<td>1</td>
</tr>
</tbody>
</table>

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Available online at www.sympnp.org/proceedings
The value of the parameter \( n \) comes out to be around 3 for muons from both momentum and angular distribution graphs. For, protons the value is around 2.88. The value of \( E_0 \) is smallest for proton where energy loss is minimum at the top of the atmosphere and highest for muons at 75º where one can expect higher energy loss at inclined direction. The value of \( E_1 \) is higher for muons at 0º, which depicts the fact that at energies around 800 GeV the power of atmospheric muon flux at sea level at 0º is around 4 as described by Gaisser parametrization. The parameter \( I_0 \) denotes the values of integrated flux at 0 degree. The proposed function gives a good description of the data of muons and can be very useful to get the integrated flux as well as the physical processes such energy loss and the shape of the spectra as a function of energy, angle and height.

![Figure 1: \( \mu \) momentum distribution at 0º at sea level.](image1)

![Figure 2: \( \mu \) momentum distribution at 75º at sea level.](image2)

![Figure 3: \( \mu \) momentum distribution at 0º at 600 m altitude.](image3)

![Figure 4: Proton flux as a function of momentum at the top of the atmosphere](image4)

![Figure 5: Muon flux as a function of zenith angle at sea level](image5)

### TABLE II: Parameters obtained from Eq. 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_0 )</td>
<td>87.93</td>
</tr>
<tr>
<td>( n )</td>
<td>3.08</td>
</tr>
<tr>
<td>( R )</td>
<td>256</td>
</tr>
<tr>
<td>( \chi^2/ndf )</td>
<td>1.0/37</td>
</tr>
</tbody>
</table>

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### References