Speed of Sound in Hadronic matter using Non-extensive Statistics

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Introduction

The evolution of the dense matter formed in high energy hadronic and nuclear collisions is controlled by the initial energy density and temperature. The expansion of the system is due to the very high initial pressure with lowering of temperature and energy density. The pressure (P) and energy density (ϵ) are related through speed of sound (c_s^2) under the condition of local thermal equilibrium. The speed of sound plays a crucial role in hydrodynamical expansion of the dense matter created and the critical behaviour of the system evolving from deconfined Quark Gluon Phase (QGP) to confined hadronic phase. There have been several experimental and theoretical studies in this direction [1]. The non-extensive Tsallis statistics [2] gives better description of the transverse momentum spectra of the produced particles created in high energy $p + p(\bar{p})$ and $e^+ + e^-$ collisions. To study, how the system behave in the presence of local temperature fluctuations, we have taken speed of sound as an observable in Tsallis statistics.

Non-extensive Statistics for Hagedorn Resonance Gas

We have used non-extensive statistics to study the c_s^2 and other thermodynamic quantities for an ideal pion gas and for Hagedorn resonance gas. The thermodynamically consistent Boltzmann distribution function for Tsallis statistics is given by,

$$f_T^B(E) = \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{1}{q-1}} \quad (1)$$

Using this distribution function, one obtains the energy density and pressure in Tsallis non-extensive form of Boltzmann distribution as:

$$\epsilon_T^B = g\int \frac{d^3p}{(2\pi)^3} E\left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}} \quad (2)$$

$$P_T^B = g\int \frac{d^3p}{(2\pi)^3}\frac{p^2}{3E}\left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}} \quad (3)$$

To include the contribution of higher resonances we introduce the q-dependent Hagedorn mass spectrum [3], which is given by:

$$\rho(m) = \delta(m - m_0) + \gamma m^{-5/2}\exp_q(m/T_c) \times\theta(m - 2m_0) \quad (4)$$

where, θ is the step function, the property of which ensures the contribution of resonance spectra above mass $2m_0$, with m_0 being the pion mass. Here $\gamma = (5 \pm 3) \times 10^{-3}$ GeV$^{3/2}$. The expression for entropy density is,

$$s(T) = \frac{\epsilon(T) + P(T)}{T} \quad (5)$$

Now one obtains the speed of sound as,

$$c_s^2 = \frac{s(T)}{C_V} \quad (6)$$

As it observed from FIG. 1, there is softening of EoS corresponding to different q dependent critical temperature.

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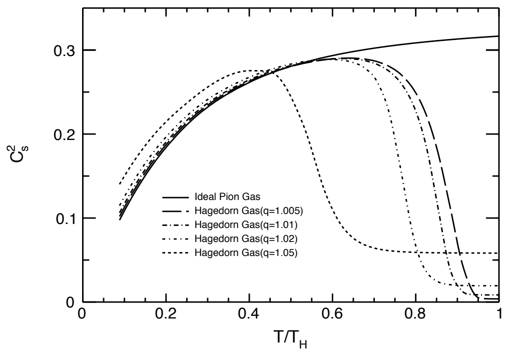

FIG. 1: Speed of sound as a function of T/T_H for ideal pion gas and for Hagedorn gas with different q-values.

Speed of Sound in a Physical Hadron Resonance Gas

The Tsallis form of Fermi-Dirac and Bose-Einstein distribution functions for hadrons are given by,

$$f_T(E) \equiv \frac{1}{\exp_q\left(\frac{E-\mu}{T}\right) \mp 1}. \qquad (7)$$

where the function $\exp_q(x)$ is defined as

$$\exp_q(x) \equiv \begin{cases} [1+(q-1)x]^{1/(q-1)} & \text{if} \quad x > 0 \\ [1+(1-q)x]^{1/(1-q)} & \text{if} \quad x \leq 0 \end{cases} \qquad (8)$$

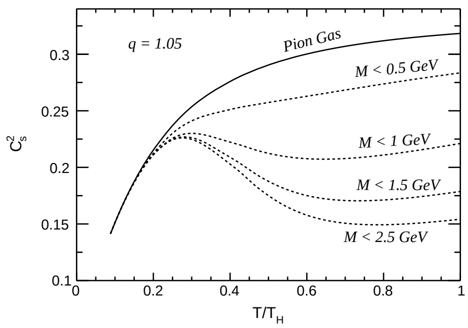

FIG. 2: Speed of sound for $q = 1.005$ for physical resonance gas with different cut-off on resonance mass.

Our findings in FIG. 2 suggest that even in q statistics, inclusion of higher mass resonances can leads to a systematic decrease of c_s^2 for a physical resonance gas.

An increase in the value of q from one shows a deviation from equilibrium statistics. To study the effect of higher values of q on the speed of sound, we have taken $q = 1.005$ to 1.15 with different mass cut-offs for the resonances, which is shown in FIG. 3. It shows that with an increasing q parameter which is related to temperature flactuation in the system, the speed of sound increases.

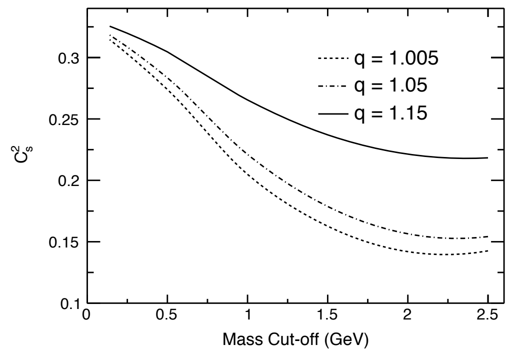

FIG. 3: Speed of sound as a function of upper mass cut-off for Hagedorn gas at T =170 MeV, with different values of q-parameter.

In the present work we study the behaviour of c_s^2 in non-extensive hadronic matter and observe that: (i) it increases with increase in q-value, (ii) it shows a q-dependent minimum at different values of temperatures referring to the softening of EoS, (iii) the q-critical temperature shows a monotonic fall with slight increase of the q-value, which is very sensitive to the degree of deviation from an equilibrated system.

References

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