The study of the energy spin relationship in ground bands of even-even nuclei in the frame work of Power Law

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Introduction

There are a number of idealized paradigms to study and understand the nuclear structure, which involve axial rotor [1], anharmonic vibrator [2] and $\gamma$-soft deformed nuclei [3, 4]. These models predict energy sequences and $B(E2)$ values.

There are several empirical formulae to express the ground state band level energies of nuclei. The simplest well-known examples are the expression for rotational spectra, $E = \frac{\hbar^2}{2\mathcal{I}} J(J+1)$, \hspace{1cm} (1)

(here $\mathcal{I}$ and $J$ are the moment of inertia and spin of the nuclei, respectively), and the Bohr-Mottelson energy expansion in powers of $J(J+1)$ for deformed nuclei [5], i.e., $E = AJ(J+1) + B(J(J+1))^2 + C(J(J+1))^3$. \hspace{1cm} (2)

Gupta et. al [6] suggested a single-term expression for ground band level energies of a soft-rotor. They replaced the concept of the arithmetic mean of the two terms used in the Bohr-Mottelson expression by the geometric mean and introduced a two-parameter formula called the power law

$E = aJ^b$ \hspace{1cm} (3)

By using Eq.(3) for any spin ($J$) the value of $b$ can be determined from the ratio

$R_J = E/E(2) = (J/2)^b$. \hspace{1cm} (4)

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Result and Discussion

Constancy of $a$

The variation of $a$ is fairly constant against $J$ for $N=42-44$ isotones (see Fig.1). At $N=36,46-48$ the variation is larger. Also, the rise or fall in $a$ is in the reverse sense as compared to the $b$.

The variation of $a$ in relation to the MI of the nuclear core is smaller as compared to the value of $b$. This implies that the dependence of energy $E(J)$ on spin $J$ is observed in the $b$ itself and MI is relatively constant in a given nuclide.

For $^{74}$Se, $^{76}$Kr and $^{78}$Sr, the $b$ values show constant straight behavior with $J$ and values lie within 1.1 and 1.2, 1.3 and 1.4 and at 1.5, respectively.

Result and Discussion

Constancy of $b$

At $N=42$, the value of $b$, for isotones of $^{76}$Se, $^{78}$Kr, $^{80}$Sr and $^{82}$Zr, lies within the range of 1.24-1.25, 1.28-1.35, 1.33-1.40 and 1.35-1.40, respectively. In case of $^{86}$Sr the value of $b$ increases up to a fixed value and then there is a sudden fall in the value.

At $N=44$, for isotones of $^{78}$Se, $^{80}$Kr, $^{82}$Sr and $^{84}$Zr nuclei, the value of $b$ lie in the range of 1.29-1.20, 1.21-1.22, 1.21-1.25 and 1.22-1.26, respectively.

At $N=46$, for isotones of $^{82}$Kr, $^{84}$Sr and $^{86}$Zr nuclei, the value of $b$ lie within the range of 1.22-1.14, 1.15-1.25 and 1.14-1.09.

At $N=48$, for isotones of $^{84}$Kr, $^{86}$Se and $^{88}$Zr nuclei the value of $b$ lie within the range of 1.24-0.93, 1.05-0.96 and 1.01-0.89, respectively.

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FIG. 1: Plot of coefficient \( a \) in various groups of isotones for \( N=42-48 \). The same symbols have been used for different elements in all parts of the figure.

FIG. 2: Plot of coefficient \( b \) in various groups of isotones for \( N=42-48 \). The same symbols have been used for different elements in all parts of the figure.

**Conclusion**

To summarize, we studied the power law, which is applicable for both deformed and soft nuclei. The formula is particularly successful in soft rotor and deformed nuclei with \( 2.8 \leq R_{4/2} \leq 3.3 \). In this work we try to apply the power law for those nuclei with \( 2.8 \leq R_{4/2} \leq 3.3 \). The power law gives good fit of the data for \( b \) and \( a \) derived either from \( 2^+ \), \( 4^+ \) or \( 6^+ \), \( 8^+ \) energy levels. It represents an alternate expression for energies in the rotation-vibration \( K^\pi=0^+ \) ground bands in deformed nuclei. The value of \( b \) is fairly constant and independent of spin \( J_\pi \), at least for low spin values \( (J_\pi < 12^+) \). The point at which the value of \( b \) exhibits a sharp drop is an indication of the shape and phase change in the nucleus. The power formula is an alternative approach to the perturbation expansion.

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**References**


**TABLE I:** Theoretical and experimental gsb levels in MeV for Mg-Zr nuclei. The fitting parameters \( a, b \) in Eq.(3) are given in MeV. The RMS factor \( \sigma \) is also given in MeV.

<table>
<thead>
<tr>
<th>Nuclei</th>
<th>( a )</th>
<th>( b )</th>
<th>( \sigma )</th>
<th>( E_{exp} )</th>
<th>( E_{th} )</th>
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<tbody>
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<td>(^{40}\text{Ar})</td>
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<td>0.8075</td>
<td>0.0633</td>
<td>1.4608</td>
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