Low spin signature inversion in the yrast band of doubly odd $^{126}$I nucleus

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Introduction:
The iodine nuclei in the mass regions ~110 - 130 have attracted considerable attention in recent years as they lie in the transitional region between spherical $^{50}$Sn nuclei and well deformed $^{56}$Ce nuclei. Recently the high spin states in $^{126}$I have been investigated by using in-beam γ-ray spectroscopy techniques [1]. The rotational level scheme of $^{126}$I has been modified and extended by adding about 80 new γ-rays and five new bands have been established by this experiment. From the experimental data the low spin signature inversion has been revealed for yrast band with configuration $\{d_{5/2} \otimes h_{11/2}\}$. This is a novel observation as it involves a low spin orbital; we have earlier reproduced the phenomenon of signature inversion in rare earth region [2] and above the rare earths for high j orbitals [3]. In this paper we have used the two quasiparticle plus rotor model (TQPRM) calculations to reproduce the signature inversion at low spin in $^{126}$I below the rare earth region. We have reported the results of our theoretical calculations for this yrast band.

Formulation:
We have used an axially symmetric two-quasiparticle plus rotor model (TQPRM) in contrast to [1] that have used a triaxial projected shell model. A detailed description of the model may be found in many papers [4]. We present here a brief description only.

The total Hamiltonian is divided into two parts, the intrinsic and the rotational,

$$H = H_{\text{int}} + H_{\text{rot}} \quad \text{.... [1]}$$

The intrinsic part consists of a deformed axially symmetric average field $H_{\text{av}}$, a short range residual interaction $H_{\text{pair}}$, and a short range neutron-proton interaction $V_{np}$, so that

$$H_{\text{int}} = H_{\text{av}} + H_{\text{pair}} + V_{np} \quad \text{.... [2]}$$

The vibrational part has been neglected in this formulation. For an axially-symmetric reflection-symmetric rotor

$$H_{\text{rot}} = \hbar^2/2\mathcal{I} (I^2/2 + H_{\text{av}} + H_{\text{ppc}} + H_{\text{tot}}) \quad \text{.... [3]}$$

$\mathcal{I}$ is the moment of inertia with respect to the rotation axis. The set of basis Eigen functions of $H_{\text{av}} + H_{\text{pair}} + \hbar^2/2\mathcal{I} (I^2/2 \otimes l^2)$ may be written in the form of the symmetrised product of the rotational wave function $D^{I\text{MK}}_{\text{MK}}$ and the intrinsic wave function $|K\alpha_p\rangle$ can be written as

$$|\text{IMK}\alpha_p\rangle = \left[\frac{2l+1}{16\pi^2(1+\delta_{K0})}\right]^{1/2} \left[D^{I\text{MK}}_{\text{MK}}(K\alpha_p) + (-1)^{l+K}\delta_{K0}R(|K\alpha_p|)\right]$$

Where the index $\alpha_p$ characterizes the configuration $\alpha_p = p_\mu p^\mu_\nu$ of the odd proton and the odd neutron. The TQPRM calculations involve many parameters. Three parameters namely Inertia parameter ($h^2/2\mathcal{I}$), Band Head Energy ($E_{\text{BH}}$) and Newby shift ($E_{\text{NH}}$) plays an important role in calculations. The Inertia parameter ($h^2/2\mathcal{I}$) is taken as 10.0keV and 10.5keV for $K_0$ and $K_1$ bands respectively and adjusted during the fitting process. The calculations require mixing of many 2qp experimentally known and unknown bands; so estimated energies of experimentally unknown bands is obtained by using semi-empirical formulation [5]. The single particle energies are estimated from neighboring odd-A nuclei where it is not known. For this nucleus the experimental data of single particle energies is almost unknown. Therefore band head energies of bands are estimated values.

Results and Discussion:
The Yrast band of $^{126}$I has a 2qp configuration $\{1/2[420]_p \otimes 7/2[523]_n\}$ with the odd proton in the $d_{5/2}$ and the odd neutron in the $h_{11/2}$ orbital. As a rule, the energetically favored signature sequence in an odd-odd nucleus has the signature $\alpha_p = (1/2(-1)^{l\otimes}) + 1/2(-1)^{l\otimes}$ while the unfavorable signature is determined by $\alpha_{\text{ad}} = 1/2(-1)^{l\otimes} + 1/2(-1)^{l\otimes}$ or $\alpha_{\text{ad}} = 1/2(-1)^{l\otimes} + 1/2(-1)^{l\otimes}$, where $j_0$ and $j_n$ are angular momentum of the valence proton and neutron respectively. Generally the levels with $\alpha_p$ are expected to lie lower in energy than the levels with $\alpha_{\text{ad}}$. But in some of the bands with certain configuration, favored signature branch lies higher in energy than the unfavorable signature at low spin. But after a certain spin $I$, called Critical spin, the favored signature branch lies lower in energy than the unfavorable signature at high spins [4]. This feature is termed as signature inversion. According to the rule, the favored signature is $\alpha_p = 0$ so even spins should be lower in energy than the odd spin states. We have gone through the experimental data[1] for yrast band.
in $^{126}$I and the experimental staggering plots of $\Delta E(I-1-1)/2I$ with angular momentum $(I)$ are shown in figure 1: we note that at low spins signature inversion occurs, where the favored signature states are located higher in energy than the unfavorable states. With increasing spin, the signature inversion ends at a spin $I=13$ in experimental plot as shown by arrow in fig.1. After this particular spin the staggering phase becomes normal [6, 7].

We have done the TQPRM calculations by incorporating rotational bands based on $d_{5/2}$ & $g_{7/2}$ proton and $h_{11/2}$ neutron configuration. The excitation energies of these bands have been estimated as the experimental data of odd-A nuclei in lower mass region is not known. The deformation parameters ($C_2$, $C_4$) used in the calculation is: (0.142, -0.007) and are taken from reference [8]. The Nilsson model parameters, $\kappa$ and $\mu$ for protons and neutrons are calculated by using the following relations respectively: $\kappa_p$ = [0.0766-0.0779(A/1000)], $\mu_p$ = [0.493±0.464(A/1000)] and $\kappa_n$ = [0.0641-0.0026(A/1000)], $\mu_n$ = [0.624-1.234(A/1000)] [9]. The calculated value of $\kappa$ and $\mu$ are 0.0668 and 0.575 for protons and 0.0638 and 0.469 for neutrons. The single particle matrix elements $\langle l_{p} \rangle$ are calculated using Nilsson wave functions [9].

The mechanism of signature inversion in $K_z=4^+$ band of $^{126}$I is due to the reverse behaviour of $K_z=0$ and $K_z=1$ bands. The $K_z=0$, {1/2}[420]$_p$ $\otimes$ 1/2[550]$_n$ band favors even spin while $K_z=1$ favors odd spin. The opposite phase of the $K_z=0$ and $K_z=1$ bands is transmitted to higher $K$ bands through the chain of Coriolis and particle-particle coupling. The reverse behaviour of $K_z=0$ and $K_z=1$ band is transmitted to $K_z=4^+$ band and producing odd-even staggering and signature inversion. The value of Newby Shift does not affect the pattern at all. Therefore only the decoupling parameter is responsible for the odd-even staggering. The fitted value of $\langle 1/2[420]|1/2[420] \rangle_p$ is -0.40339 (-1.78534) and $\langle 1/2[431]|1/2[431] \rangle_n$ is -2.33463 (2.89349) where Nilsson matrix value is given in the parenthesis. These matrix elements play an important role to reproduce the odd-even staggering and signature inversion at low spin region in $^{126}$I. We have not done any actual fitting of experimental data. However the point of signature inversion has been shifted by one unit as compared with the experimental data. Figure 1 shows the plot between $\Delta E(I-1-1)/2I$ (keV) versus angular momentum $(I)$ for the calculated and experimental values separately.

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**Fig. 1:** Results of TQPRM calculations are plotted for $\Delta E(I-1-1)/2I$ (keV) vs. $I$ for yrast band in $^{126}$I. Point of signature inversion ($I_c$) is shown by arrow. Results from the calculations compared with the experimental data for the two-quasiparticle rotational bands. Note that the scale is different in two plots.

**Conclusion:**

The results of TQPRM calculations for yrast band $K_z=4^+$, {1/2}[420]$_p$ $\otimes$ 1/2[550]$_n$ in doubly odd $^{126}$I are presented. The low spin signature inversion ($I_c$) is well reproduced by TQPRM calculations. However the point of signature inversion is shifted by one unit but the feature is well reproduced. It is very much clear that the phenomenon of signature inversion which was observed in high-$j$ orbitals of rare earths is also present in low-$j$ orbitals and the same mechanism can reproduce it.

**References:**


