Detrended fluctuation analysis in multiparticle production

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The multifractal detrended fluctuation analysis (MF-DFA) introduced in [1] is found to be a highly successful method in analyzing nonstationary stochastic processes. So far the method has been applied to different areas of statistical analysis, for instance see [2] and the references therein. In this paper we apply the technique to the pseudorapidity (η) distribution of shower tracks coming out of 28Si-Ag(Br) events at an incident energy of 14.5A GeV. Each event has a shower track multiplicity n > 50. We compare the experiment with the prediction of the Ultra-relativistic Quantum Molecular Dynamics (UrQMD) [3].

In the MF-DFA formalism first a “profile” function Y is to be determined out of the data points x_k as:

\[ Y(i) = \sum_{k=1}^{i} [x_k - \langle x \rangle], \text{ } i = 1, \ldots, N. \]  

Then the profile Y(i) is divided into N_s \equiv \text{int}(N/s) segments of equal length s. Then the variance of each segment p with respect to the local trend:

\[ F^2(p, s) = \frac{1}{s} \sum_{i=1}^{s} [Y[(p-1)s+i] - y_{p}(i)]^2, \]  

is obtained. Here y_p(i) represents the local trend for the segment p. We consider a linear trend of the event-wise local particle density i.e., x_k = dn/dη. The density distribution plot (dn/dη against η) for a typical high multiplicity event is shown in Fig. 1. Finally, the qth order MF-DFA function is defined as:

\[ F_q(s) = \left\{ \frac{1}{N_s} \sum_{p=1}^{N_s} [F^2(p, s)]^{q/2} \right\}^{1/q} \]  

For any q ≠ 0. For q = 0 the definition is modified as:

\[ F_0(s) = \exp \left\{ \frac{1}{2N_s} \sum_{p=1}^{N_s} \ln[F^2(p, s)] \right\}. \]  

If the series x_k is a fractal one then F_q(s) for large s and for all q would exhibit a power-law scaling behaviour like: F_q(s) \sim s^{h(q)}. In general, for a multifractal series the exponent h(q) depends on q while for a monofractal series it is expected to be independent of q, i.e., h(q) = H, the Hurst exponent [4]. Moreover, for stationary series h(2) = H [4]. Thus, one can distinguish the function h(q) as the generalized Hurst exponent, which is related to the multifractal scaling exponent τ(q) as τ(q) = q h(q) − 1. The multifractal singularity spectrum f(α) is determined via a Legendre transformation: f(α) = qα − τ(q), where α = τ(q).

Since the method, originally developed for a nonstationary time series of effectively infinite length, is applied to a series of finite length (n ≥ 50), we average the MF-DFA fluctuation function F_q(s) over the total number of events (N_{ev} = 158) in our sample. Fig. 2 shows the event averaged MF-DFA fluctuation functions.

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FIG. 2: Log-log plots of the event averaged MF-FDA fluctuation functions $F_q(s)$ with scale $s$.

$F_q(s)$ plotted against the scale $s$ for several values of $q$. As expected both the experiment and the UrQMD generated plots follow the power-law type of scaling. The exponent $h(q)$ are calculated from the linear fits to the data points for $q = -5$ to $+5$. The order dependence of the generalized Hurst exponents is shown in Fig. 3(a) and the corresponding $\tau(q)$ exponent spectra and the multifractal spectra are shown, respectively, in Fig. 3(b) and 3(c).

However, the complete $f(\alpha)$ spectrum could not be obtained, because of its unusual behaviour in the $q < 0$ region, as is also seen in a similar analysis [5]. The observed nonlinearity in the $h(q)$ and $\tau(q)$ spectra and the concave nature of the $f(\alpha)$-spectra are clear signatures of multifractality in the $\eta$-distribution of the event samples analyzed. The experiment and the UrQMD exhibit more or less similar trends but the degree of multifractality is a little weaker in the simulation. The results of this analysis are almost consistent with those of our previous multifractal analysis using a different technique [6].

In summary, we have applied the multifractal detrended fluctuation analysis in order to characterize the $\eta$-distribution of charged particles emitted from $^{28}$Si-Ag(Br) collisions at an incident energy of 14.5A GeV. The results of our analysis show multifractal nature in the $\eta$-distribution for both the experiment and the UrQMD simulation. From our preliminary results it is expected that the MF-DFA method will reliably characterize the multifractal pattern of the phase-space distribution in high-energy heavy-ion collisions.

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References


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