Chaos in Strong Nuclear Interactions

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Introduction

Since last few decades, the issue of quark confinement is a challenging task which is yet to be solved completely. Many models of the field theory of quarks and gluons are proposed to understand the confinement problem and other closely related aspects [1–3]. Quantum chromodynamics (QCD) is quite successful to address the problems associated with confinement mechanism [4, 5]. In this work, we have tried to further explore the chaotic field dynamics of QCD in view of running nature of strong coupling constant (αs).

Chaotic Field Analysis of QCD

The non-linearity present in the Yang-Mills equation makes the QCD lagrangian non-integrable [6]. This non-integrability cause chaos in quark-glueon system. Such non-integrability can be transcribed in an analogous way to the non-integrability in high energy regime. However, in low energy regime, it becomes very complicated. The chaotic field analysis of QCD has shown some interesting results to understand the confinement problem and other closely related aspects [1–3]. Quantum field theory of quarks and gluons are proposed to describe in terms of the energy eigenvalues (E1 ≤ E2 ≤ E3...En ≤ ...) of the potential discussed above. The distinction between regular and irregular spectral sequences

The quantum spectrum of such systems may be derived from this Hamiltonian.

\[ H = \frac{1}{2}(q_1^2 + q_2^2) + \frac{(1 - \beta)}{12}(q_1^4 + q_2^4) + U, \]

with β = 1, the Hamiltonian (2) reduces to the problem with the non-central potential given by Eq(1) while for β = 0 it is an integrable one. In order to make an analogy of non-central potential (1) with the non-Abelian gauge theory to study the non-linearity in strong nuclear interactions, the Yang-Mills equation (without quarks) for the simple choice of non-Abelian gauge group SU(2) can be written as follows,

\[ \partial_\mu F^a_{\mu\nu} + g_s \epsilon^{abc} A^b_\mu F^c_{\mu\nu} = 0, \]

where the gauge field strength is given by,

\[ F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s \epsilon^{abc} A^b_\mu A^c_\nu, \]

where dots show time derivatives. If we consider the Hamiltonian given below,

\[ H_A = \frac{1}{2}(\dot{A}^a_v)^2 + \pi \alpha_s \{ (A^a_v A^a_v) - (A^a_v A^a_v)^2 \}, \]

where, \( \alpha_s = g_s^2/4\pi \) is the strong coupling constant. The equation of motion given by Eq(5) can also be derived from this Hamiltonian. The quantum spectrum of such systems may be described in terms of the energy eigenvalues (E1 ≤ E2 ≤ E3...En ≤ ...) of the potential discussed above. The distinction between regular and irregular spectral sequences

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may be analysed with the distribution of nearest neighbor spacing with the energy difference $\Delta E_n = E_{n+1} - E_n$ between $(n+1)$th and $n$th level [5]. The probability $P(Y)$ of a given spacing $\Delta E$ (between neighboring levels in the neighborhood of energy level E) for such chaotic potential has been found to be very close to the Wigner distribution as given below [9],

$$P(Y) = \frac{\pi}{2} Y \exp\left(\frac{-\pi Y^2}{4}\right),$$

(7)

where, $Y = \Delta E / <\Delta E>$ being the mean level spacing. The running nature of the strong coupling constant ($\alpha_s$) may thus be used to explore the regularities and irregularities at different energy (length) scales to explain the quark confinement and quark-gluon plasma (QGP) formation. The strong coupling constant ($\alpha_s$) observed through deep inelastic scattering (DIS) experiments is ranged as, $0.1 < \alpha_s < 0.5$. The perturbative QCD domain is defined in range $0.1 < \alpha_s < 0.2$ and non-perturbative QCD domain in range $0.2 < \alpha_s < 0.5$. If we compare the role of $\alpha_s$ in Eq(6) with the $\beta$ in Eq(2), it becomes clear that the strong coupling constant may become helpful in describing the chaotic view in different energy regimes of strong interactions.

**Results and Conclusions**

If one considers the extreme values of $\alpha_s$ in both the regimes (i.e., $\alpha_s \sim 0$ in perturbative sector and $\alpha_s \sim 1$ in non-perturbative sector), then it is evident from Eq(6) that the non-integrable potential is present only in the regime of non-perturbative sector. The chaos is, therefore, present when the quarks are confined while the absence of non-linearity, i.e., $\alpha_s \sim 0$, leads to the asymptotically free states (formation of QGP) perhaps having no chaos. Even the presence of sources (the dynamical quarks) in the present formulation may not influence the observations in terms of $P(Y)$ because the second term in Eq(6) remains intact. It is worth to notice that the complexity and universality of behavior were earlier thought to be non-compatible. But the chaos has showed the randomness and universality could coexist in different phenomena in nature as evident by the present analysis of strong nuclear interactions.

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**References**


