Two photon decays of charmonia
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Introduction
Recently, many new charmonium like states such as the X, Y, Z, X′, X′′ etc. have been experimentally discovered [1]. Some of these states were predicted to be \(\bar{Q}Q\) states but their possibility to be exotic hadrons cannot be ruled out[2, 3]. Correct identification of these states with proper \(J^{PC}\) values poses a challenge.

Two photon decay is an important property in the study of Quarkonia as it provides additional information regarding the quark-antiquark dynamics. Successful prediction of two photon decay widths can be used to assign correct \(J^{PC}\) values to various states and hence can be employed for further classification of the states.

Methodology
For the present study we employ the hamiltonian[4]
\[
H = \sqrt{p^2 + m_Q^2} + \sqrt{p^2 + m_{\bar{Q}}^2} + V(r);
\]
\[
V(r) = -\frac{4\alpha_S}{3r} + Ar + V_0,
\]
where \(\alpha_S\) is the running strong coupling constant, \(A\) is a potential parameter and \(V_0\) is a constant. We expand the hamiltonian up to order \(p^6\). The value of the QCD coupling constant \(\alpha_S\) is determined as in ref. [5]. The quark mass employed in the present calculation is \(m_c = 1.55\) GeV. We use variational method in which the trial wave function is a Gaussian given by
\[
R_{nl}(\mu, r) = \frac{\mu^3}{2 (n-1)!} \Gamma(n+l+1/2) \mu (\mu r)^l e^{-\mu^2 r^2/2} L_{n-1}^{l+1/2}(\mu^2 r^2);
\]
\(\mu\) is a variational parameter and \(L\) is Laguerre polynomial. We set the value of \(A = 0.175\) GeV². The variational parameter \(\mu\) is determined by making use of the virial theorem[6]. With this value of \(\mu\) spin-averaged(SA) mass of charmonia is obtained by
\[
H\psi = E\psi.
\]
The ground state SA mass is matched with the experimental SA mass in order to determine the constant \(V_0\). The value of constant \(V_0 = -0.236\) GeV.

We add separately (in Eq.4) the spin-dependent part to the usual one gluon exchange potential between the quark antiquark for computing the hyperfine and spin-orbit shifting of the low-lying S wave states. The form of the spin-dependent potential can be found in ref [7].

Two photon decays
In the non-relativistic limit, the two-photon decay widths of \(^1S_0\), \(^3P_0\), and \(^3P_2\) can be written as [8]
\[
\Gamma_{NR}(^1S_0 \to \gamma\gamma) = \frac{3\alpha^2 e_c^4 |R_{ns}(0)|^2}{m_c^2},
\]
\[
\Gamma_{NR}(^3P_0 \to \gamma\gamma) = \frac{27\alpha^2 e_c^4 |R_{np}(0)|^2}{m_c^2},
\]
\[
\Gamma_{NR}(^3P_2 \to \gamma\gamma) = \frac{81\alpha^2 e_c^4 |R_{np}(0)|^2}{5m_c^2}.
\]
TABLE I: Two photon decay widths in charmonia.

<table>
<thead>
<tr>
<th>Transition</th>
<th>Mass (GeV)</th>
<th>µ (GeV)</th>
<th>$R_{nS}(0)$ (GeV$^{3/2}$)</th>
<th>$R_{nP}(0)$ (GeV$^{5/2}$)</th>
<th>Width (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>This work</td>
<td>Expt.[1]</td>
<td>Ref[9]</td>
</tr>
<tr>
<td>$1^1S_0 \to \gamma\gamma$</td>
<td>2.984</td>
<td>0.655</td>
<td>0.797</td>
<td>8.34</td>
<td>3.86</td>
</tr>
<tr>
<td>$2^1S_0 \to \gamma\gamma$</td>
<td>3.665</td>
<td>0.460</td>
<td>0.382</td>
<td>1.92</td>
<td>&lt; 5</td>
</tr>
<tr>
<td>$3^1S_0 \to \gamma\gamma$</td>
<td>4.111</td>
<td>0.405</td>
<td>0.283</td>
<td>1.05</td>
<td>0.88</td>
</tr>
<tr>
<td>$1^3P_0 \to \gamma\gamma$</td>
<td>3.455</td>
<td>0.498</td>
<td>0.215</td>
<td>2.27</td>
<td>2.33</td>
</tr>
<tr>
<td>$2^3P_0 \to \gamma\gamma$</td>
<td>3.930</td>
<td>0.420</td>
<td>0.140</td>
<td>0.97</td>
<td>0.99</td>
</tr>
<tr>
<td>$3^3P_0 \to \gamma\gamma$</td>
<td>4.314</td>
<td>0.385</td>
<td>0.113</td>
<td>0.63</td>
<td>0.64</td>
</tr>
<tr>
<td>$1^3P_2 \to \gamma\gamma$</td>
<td>3.554</td>
<td>0.498</td>
<td>0.215</td>
<td>0.60</td>
<td>0.09</td>
</tr>
<tr>
<td>$2^3P_2 \to \gamma\gamma$</td>
<td>4.008</td>
<td>0.420</td>
<td>0.140</td>
<td>0.26</td>
<td>0.04</td>
</tr>
<tr>
<td>$3^3P_2 \to \gamma\gamma$</td>
<td>4.388</td>
<td>0.385</td>
<td>0.113</td>
<td>0.17</td>
<td>0.02</td>
</tr>
</tbody>
</table>

The first-order QCD radiative corrections to the two-photon decay rates can be accounted for as[8]

$$\Gamma(1^1S_0 \to \gamma\gamma) = \Gamma_{NR}(1^1S_0 \to \gamma\gamma) \times \left[1 + \frac{\alpha_S}{\pi} \left(\frac{\pi^2}{3} - \frac{20}{3}\right)\right]$$  \hspace{1cm} (8)

$$\Gamma(3^1P_0 \to \gamma\gamma) = \Gamma_{NR}(3^1P_0 \to \gamma\gamma) \times \left[1 + \frac{\alpha_S}{\pi} \left(\frac{\pi^2}{3} - \frac{28}{9}\right)\right]$$  \hspace{1cm} (9)

$$\Gamma(3^3P_2 \to \gamma\gamma) = \Gamma_{NR}(3^3P_2 \to \gamma\gamma) \times \left[1 - \frac{16\alpha_S}{3\pi}\right]$$  \hspace{1cm} (10)

In calculating these decay widths we have made use of the spectroscopic parameters obtained from the present model.

Results and discussion

The two-photon decay widths calculated within the present framework are reported in table I. Reliable experimental measurements are few. It can be observed that the decay widths obtained with QCD correction are in satisfactory agreement with experimental measurements for the states $1^1S_0$ and $1^3P_0$. While our prediction for decay width of the state $1^3P_2$ is underestimated. Further experimental measurements are required to verify the model.

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References