New form of Geiger-Nuttall law in $\alpha$-radioactivity

Ramkrishna Paira* and Basudeb Sahu

Department of Physics, North Orissa University, Baripada-757003, INDIA

Introduction

The Geiger-Nuttall (GN) law is a famous age old formula which explain the measured values of half-life of $\alpha$-decay with a mathematical characteristic of linear variation of logarithmic half-life with inverse square root of Q-value in $l = 0$ state of transition or decay. Geiger-Nuttall plots for $\alpha$-decay are expressed as $\log T_{1/2}=\alpha Q^{1/2}+b$, since different expression have been proposed [5, 7] to calculate $\log T_{1/2}$ from A, Z and the Q-value. The adjustment of the formula coefficients $a$ and $b$ is realized on the total experimental results of $\alpha$-decay half-lives. To explain half-life of emitted particle carrying some angular momenta, we derive an analytical expression for the decay half-life akin to GN law by considering the unstable parent nucleus as a quantum two-body system of the ejected $\alpha$-particle and the daughter nucleus exhibiting resonance scattering phenomena under the combined effect of nuclear, Coulomb and centrifugal forces. The formula coefficients in our expression for the nuclear, Coulomb and centrifugal forces. The result of the above expression gives values of half-life of $\alpha$-decay up to the outside Coulomb Hankel outgoing spherical wave $f_c(kr) = G_l(\eta, kr) + iF_l(\eta, kr)$ such that

$$u(r) = N_0[\frac{G_l(\eta, kR)}{iF_l(\eta, kR)}],$$  \hspace{1cm} (1)

where $R$ is the radial position outside the range of the nuclear field.

For a typical $\alpha$-nucleus system with $\alpha$-particle as the projectile and the daughter nucleus as the target, let $\mu$ represent the reduced mass of the system and the wave number $k = \sqrt{2\mu E}$ and $\eta$ stands for the Coulomb parameter

$$\eta = \sqrt{\frac{2\mu e^2}{\hbar^2}}.$$  \hspace{1cm} (2)

With this the mean life $T$ (or width $\Gamma$) of the decay is expressed in terms of amplitude $N_0$ as

$$T = \frac{\hbar}{\Gamma} = \frac{\mu}{\hbar k} \left| \frac{1}{N_0} \right|^2.$$  \hspace{1cm} (3)

Since the wave function $u(r)$ decreases rapidly with radius outside the daughter nucleus, it can be normalized by requiring that $\int_0^R |u(r)|^2 dr = 1$. Further, using the fact that for a value of radial distance sufficiently large, the value of $G_l(\eta, kR)$ is very large as compared to $F_l(\eta, kR)$ by several order of magnitude, the $T$ of Eq. (2) is expressed as

$$T = \frac{\mu}{\hbar k} \left| \frac{G_l(\eta, kR)}{P} \right|^2,$$  \hspace{1cm} (4)

where

$$P = \frac{\int_0^R |u(r)|^2 dr}{\int_0^R |u(r)|^2 dr}.$$  \hspace{1cm} (5)

Result of the above expression gives values of mean life $T$ or half-life $T_{1/2}=0.693 T$ of the decay of the charged particle carrying angular momentum $l$ with Q-value equal to the resonance energy E. In this article, we simplify the formula (3) and put it in the well known linear

---

*Electronic address: krishna.nou@gmail.com

Available online at www.sympnp.org/proceedings
form of GN law for the variation of $\log T_{1/2}$ as a function of Q-value of the particle emitted with some amount of angular momentum $l$.

In the special case of Coulomb-nuclear problem, there are specific values of $\eta$ and $\rho = kR$ for which the Coulomb Hankel function $G_l$ can be expressed in some simple mathematical form giving quite accurate result. Using $2\eta > \rho$ with $l \geq 0$ [8], and after simplification we get the final formula for $\log T_{1/2}$ is given by

$$\log T_{1/2} = a' \chi' + b' \rho' + c + d, \quad (5)$$

$$\chi' = ZeZd\sqrt{2} \, \rho' = \sqrt{AZcZd(A_1^{1/3} + A_d^{1/3})},$$

where $A = \frac{m}{A_1^{1/3} + A_d^{1/3}}$. The constants $a'$, $b'$, $c$, and $d$ are expressed as

$$a' = 2a_0c^2/2m/\ln 10, \quad b' = -b_f\sqrt{2me^2r_0}/h\ln 10,$$

$$c = \ln c_f/\ln 10, \quad d = -\left(\frac{a_2}{2}\right) + \frac{8}{27}r_0 + \cdots + \frac{\rho^2}{a_0^2 + \rho^2}/\ln 10 + \ln M_l/\ln 10,$$

$$b_f = 2 + a_0 - 2a_1 + (\frac{\sqrt{3}}{2} + a_1 - 2a_3)t^{1/2} + \left(\frac{a_0}{\sqrt{3}} + \frac{a_1}{2} + a_2 - 2a_3 - \frac{3}{2}\right)t + \left(\frac{a_0}{\sqrt{3}} + \frac{a_1}{2} + a_2 - \frac{3}{2}\right)^{3/2} + \left(\frac{a_0}{\sqrt{3}} + \frac{a_1}{2} + a_2 - 1\right)^2 + \left(\frac{a_0}{\sqrt{3}} + \frac{a_1}{2} + a_2 - \frac{3}{2}\right)^{1/2} + \left(\frac{a_0}{\sqrt{3}} + \frac{a_1}{2} + a_2 - \frac{3}{2}\right)^{3/2}.$$

$$c_f = \left[\frac{0.603}{\sqrt{3}}\right]\left(\frac{m}{2\pi}\right)^{1/4}A_1^{1/3} + A_d^{1/3}r_0/ZcZd \quad 10^{-23}.$$

$$t = \rho/2\eta < 1, \quad a_0 = 1.5707288, \quad a_1 = -0.2121144, \quad a_2 = 0.074240, \quad a_3 = -0.018729 [8],$$

$$\sqrt{M_l} = 1 + \frac{[2(2+1)+1]^2}{6(2+1)!} + \frac{[2(2+1)+1]^2-1![2(2+1)+1]^2-9}{2!6(2+1)!},$$

nucleon mass $m = 931.5$ MeV, square of electronic charge $e^2 = 1.4398$ MeV fm, $h = 197.329$ Mev fm, and radial distance parameter $r_0 = \frac{\rho}{A_1^{1/3} + A_d^{1/3}}$ expressed in fm unit. The constant $d'$ is $l$-dependent and it helps estimate the value of decay life-time of particles pushed out with angular momentum $l > 0$.

We use $R = 9.5$ fm and $P = 10^{-3}$ for all types of $\alpha$-daughter nuclei. The radial distance $R = 9.5$ fm is a distance over which the value of amplitude of the resonant wave function reduces to a small value of $\sqrt{2}$. We apply the formulation to several cases of

$\alpha$-daughter nuclei. The experimental data are explained quite well by our calculated results in the situations of $l = 0, 1, 2, 3, 5$, shown in Table 1.

\begin{table}[h]
\centering
\begin{tabular}{c c c c c}
\hline
$Q$(MeV) & $\log T_{1/2}/s$ & $T_{1/2}/ms$ & $\log T_{1/2}/s$ & $T_{1/2}/ms$
\hline
3.690 & 3.89 & 3.166 & 2.652 & 15.03 & 14.80
5.741 & 2.690 & 2.489 & 7.060 & -1.960 & -2.347
7.595 & -0.280 & -1.539 & 5.748 & 12.120 & 9.337
6.185 & 3.1120 & 8.421 & 8.442 & 5.200 & 2.574
\hline
\end{tabular}
\caption{Comparison of experimental values [6] of $\alpha$-decay half-lives and results of present calculation obtained by using formula (5).}
\end{table}

Acknowledgments

This work is supported by the Board of Research in Nuclear Sciences (BRNS), Department of Atomic Energy (DAE), Mumbai, India, via Research Grant No. 2011/37P/28/BRNS/1781.

References