Semi empirical Formula for neutrinoless double beta decay

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Introduction

Double beta decay is a radioactive decay process where a nucleus releases two beta rays as single process. Here two neutrons in the nucleus are converted into two protons, and two electrons and two electron antineutrinos are emitted. In order for beta decay to be possible, the final nucleus must have larger binding energy than the original nucleus. Double beta decay is difficult to study in most practically interesting cases, because both beta decay and double beta decay are possible, with probability favouring beta decay; the rarer double beta decay process is masked by these events. Thus double beta decay is usually studied only for beta stable nuclei. Only ten of them were observed to decay via the two neutrino mode. Two different and complementary methods are mainly used to calculate NME’s for neutrinoless double beta decays. One is the family of quasi particle random phase approximation (QRPA). This method has been used by different groups and variety of techniques is employed with results for most of the possible emitters [1]. This work concerned to the alternative, the interacting shell model (ISM) [2]. It has been shown that as the difference in deformation between parent and daughter grows, the NME’s of both the neutrinoless and two neutrino mode decreases rapidly. In the present work we would like to propose a semi empirical formula for computing the neutrino less double beta decay half lives.

Semi empirical formula

In the standard scenario, when 0νββ decay process occurs by exchange of light Majorana neutrinos between two nucleons inside the nucleus, and in the presence of left handed weak interactions, the life time expression can be written as a product of three factors, given as

\[ T_{1/2}^{-1} = G^{(0)}_{0ν} |M_{0ν}|^2 \left( \frac{\langle m_ν \rangle}{m_e} \right)^2 \]  

(1)

Where \( G_{0ν} \) is the phase space factor for this decay mode, \( \langle m_ν \rangle \) is the effective neutrino mass parameter, \( m_e \) is the electron mass and \( M_{0ν} \) are the NMEs depending on the nuclear structure of the nuclei involved in the decay.

The phase space factor is depending on the energy decay \( Q_{0ββ} \) and nuclear charge \( Z \). Figures 1 and 2 represent the plot of phase space factor versus \( ZQ_{3}^3 \) and \( Z^2Q_{6}^6 \) for various isotopes undergoing neutrino less double beta decay. The phase space factor is taken from ref [3]. From the observed dependence of \( ZQ_{3}^3 \) and \( Z^2Q_{6}^6 \) of the plots we have developed a semi empirical formula for the phase space factor. Using the \( ZQ_{3}^3, Z^2Q_{6}^6 \) and \( Z^3Q_{9}^9 \) variables, a new formula is obtained by making least-squares fit to the data and is given as

\[ G^{(0)}_{0ν} = a(ZQ_{3}^{3}) - b(Z^2Q_{6}^{6}) + c(Z^3Q_{9}^{9}) + d \]  

(2)

The constants are, \( a = 2.48904E-26 \), \( b = 2.20171E-38 \), \( c = 9.95199E-51 \), \( d = 1.11378E-15 \)

Figure 3 represents the plot connecting the comparison of computed phase space factor with those obtained from Ref [3] for neutrinoless double beta decay from various isotopes. It is found from the plot that the computed values are in better agreement with the values of Robertson[3].

Due to the two-body nature of the transition operator, the NMEs can be also expressed as a sum of products of two-body transition densities (TBDTs) and matrix elements of the two-body transition operators for two-particle states. We have seen a \( Z^{1/3} \) dependence of nuclear matrix element and a new formula is obtained by making least-squares fit to the data taking from the ref [4] and is given as,
The constants are, 
\[ a = -9.49274 \times 10^6, \quad b = 6.65787 \times 10^7, \]
\[ c = -2.33125 \times 10^8, \quad d = 4.07518 \times 10^8, \]
\[ e = -2.84509 \times 10^8, \quad f = 540571.09879. \]

**Results discussion and conclusion**

Table 1 shows the comparison of computed half-life time with the experimental data for neutrino less double beta decay. It is found from the table that our present formula predictions are in good agreement with the experimental values.

In the present calculation \( \langle m_r \rangle \) is taken as 50meV and is obtained from Rodin et al [4].

We have also computed the standard deviation of calculated half lives from the experimental values and other nuclear models and is found that the present value= 3.883156, QRPA = 3.578705, Truncated Shell Model=3.2091 and NSM=4.062309. It is found from the values that the present formula is better than NSM and slight greater than QRPA and Truncated Shell Model, but the present formula is very simple for computation and the other two models are complicated.

**Table 1:** The Q values, \( M_{0\nu} \) and half lives of double beta decay with the present formula and is compared with the experimental values.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Q value (KeV)</th>
<th>( M_{0\nu} )</th>
<th>Half life (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{48})Ca</td>
<td>4272</td>
<td>859.2</td>
<td>&gt;5.8E22</td>
</tr>
<tr>
<td>(^{76})Ge</td>
<td>2039</td>
<td>2.90</td>
<td>&gt;1.9E25</td>
</tr>
<tr>
<td>(^{82})Se</td>
<td>2995</td>
<td>3.04</td>
<td>&gt;3.6E23</td>
</tr>
<tr>
<td>(^{96})Zr</td>
<td>3350</td>
<td>3.92</td>
<td>&gt;9.2E21</td>
</tr>
<tr>
<td>(^{100})Mo</td>
<td>3034</td>
<td>3.48</td>
<td>&gt;1.1E24</td>
</tr>
<tr>
<td>(^{110})Cd</td>
<td>2814</td>
<td>2.37</td>
<td>&gt;1.7E23</td>
</tr>
<tr>
<td>(^{128})Te</td>
<td>866</td>
<td>2.53</td>
<td>&gt;1.5E24</td>
</tr>
<tr>
<td>(^{130})Te</td>
<td>2527</td>
<td>2.53</td>
<td>&gt;2.8E24</td>
</tr>
<tr>
<td>(^{136})Xe</td>
<td>2458</td>
<td>2.68</td>
<td>&gt;4.5E23</td>
</tr>
<tr>
<td>(^{150})Nd</td>
<td>3371</td>
<td>1.37</td>
<td>&gt;1.8E22</td>
</tr>
<tr>
<td>(^{160})Gd</td>
<td>1730</td>
<td>3.09</td>
<td>&gt;1.3E21</td>
</tr>
</tbody>
</table>

![Fig. 1](image1.png)  
**Fig. 1** The plot of phase space factor versus \( ZQ^3 \) for various isotopes undergoing neutrinoless double beta decay.

![Fig. 2](image2.png)  
**Fig. 2** The plot of phase space factor versus \( Z^2Q^6 \) for various isotopes undergoing neutrinoless double beta decay.

![Fig. 3](image3.png)  
**Fig. 3** The plot connecting the comparison of computed phase space factor with those obtained from Ref [3] for neutrinoless double beta decay.

**References**