Introduction

New shapes and structure of nuclei have been fascinating to nuclear physicists since long, however these have renewed attraction due to advancement in the experimental facilities and increasing accessibility to nuclei lying at extremes in the nuclear landscape. The advancements in experimental nuclear physics facilities have led our experimental access to many new shapes and structures in the nuclear physics, which exists in nuclear chart but do not occur naturally. This has been quite helpful in developing our understanding for some of the fundamental questions of astrophysics as well as nuclear physics and bridging the both. Because of these various possibilities, a thorough study of the stability of a bubble nucleus is called for.

Possibility of the formation of bubble nuclei has been explored through different nuclear models and in different mass regions. The major concern in all these investigations has been to find out the reason for anomalous depletion of density in the center part, especially in the spherical cases. The main mechanism for the formation of bubble nuclei is the lack of particles in the center of nucleus which causes the s levels to be less bound than observed in the usual cases with the uniform density distribution. If the particles rise high enough in energy highest s level will be empty, hence depleting the central density of particles. Subsequently, the lower s levels being less bound will increase the radius and decrease the central part of density. This may be also interpreted as s-d orbital inversion as discussed by Zhao et al [1] and E Khan et al [2]. It has been also studied that at high nuclear temperature, the surface-tension coefficient may decrease resulting in pushing the nuclear matter outward, leading to the formation of toroidal and bubble nuclei [3]. In recent past, a few experimental accessible cases, e.g. $^{22}$O and $^{34}$Si have picked up a large attention due to significant depletion of nuclear density in the center part of nucleus. Here we have first time explored the possibility of observing deformed bubble nuclei and mechanism for the same.

Mathematical Formalism

Relativistic Mean Field (RMF) model which incorporate the spin-orbit naturally, has been successfully used to understand and explain many nuclear features in the different mass region of the nuclear mass table. There exist a number of parameter sets for solving the standard RMF Lagrangian [4]. As a test case, We calculated binding energy and ground state quadrupole deformation for $^{28}$Si using different parameter sets (ref. to Fig. 1). From Fig. 1, one can see there are many parameter sets, which reproduce the binding energy up to similar accuracy but calculated ground state deformation vary significantly. So, to optimize the quality of our results we have used NL065 parameter set throughout this work, which reproduces binding energy as well as deformation satisfactorily for $^{28}$Si and for other nuclei also in light mass region. We find that this parameter set also gives better density distribution agreement in comparison to observed experimental data, especially in comparison to results obtained with other parameter sets, namely NL3, NL3*, NL-SH.

Result and Discussion

We have calculated various ground state observable and depletion fraction [5] for A=34. From fig. 2, one can see that the maximum depletion fraction occurs for $^{34}$Si (Z=14) and
then the second case with the significant depletion fraction is $^{34}$Ca ($Z=20$), which are spherical cases. This motivated us to take detailed study for $N$ or $Z=14$ cases, and we found that $^{24}$Ne ($N=14$), $^{32}$Si ($Z=14$) and $^{32}$Ar ($N=14$), which are deformed too, show significant bubbleness, as tabulated in table 1.

Table 1: Depletion fraction, DF (in percent) for the neutron and proton density, and the deformation of bubble for $^{24}$Ne, $^{32}$Si, $^{32}$Ar and $^{34}$Si. (DF)$_n$ and (DF)$_p$, are the depletion fraction along the perpendicular axis, depletion along the symmetry axis, $z$ respectively. $\delta_{\text{bubble}}$ is the deformation of the bubble.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Neutron (DF)$_n$, (DF)$_p$</th>
<th>Proton (DF)$_n$, (DF)$_p$</th>
<th>$\delta_{\text{bubble}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{24}$Ne</td>
<td>13 11</td>
<td>21 18</td>
<td>0.25</td>
</tr>
<tr>
<td>$^{32}$Si</td>
<td>4 0</td>
<td>17 14</td>
<td>-0.25</td>
</tr>
<tr>
<td>$^{32}$Ar</td>
<td>15 11</td>
<td>4 0</td>
<td>-0.25</td>
</tr>
<tr>
<td>$^{34}$Si</td>
<td>17 4</td>
<td>26 25</td>
<td>-0.03</td>
</tr>
</tbody>
</table>

We find that the origin of the bubble structure in these cases is closely related to the filling of 0d-1s orbital [6]. We observe that with all particles in 0d-orbital and no particle occupying the 1s orbital, bubble nuclei is formed, which disappears with the filling of 1-s orbital. Deformation indeed causes shell mixing and one expects the bubble structure to decrease with increasing deformations for 14 particles. These results will be presented and discussed in the conference in detail, which is a first of its kind on the subject.

The possibility for existence of deformed bubble nuclei has been investigated in conjunction with RMF model. We find that $^{24}$Ne, $^{32}$Si and $^{32}$Ar may be considered as potential cases for deformed bubble nuclei, all with significant central density depletion. As RMF based approaches determine the ground state observables and nuclear density distributions quite successfully, these could be further used for better understanding of bubbleness through exploring other bubble cases in higher mass region. Such studies also need to be complimented by shell model or similar calculations where more correlations could be include and effect of mixing may be seen in better manner.

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Fig. 1 Binding energy and ground state quadrupole deformation ($\beta_2$) for $^{28}$Si, as calculated with different parameter sets.

Fig. 2 Total Depletion fraction, DF (in percent) for $A=34$ cases.

References