Microscopic - Macroscopic Description of Ground State Nuclear Masses

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Introduction

The study of ground state nuclear masses has been a subject of interest for a number of years. A variety of different models have been reported in the literature so far [1–6] that describe the ground state masses of the nuclei spanning the entire periodic table with varying degrees of precision. The idea behind the present investigation is to develop a mic - mac model for ground state nuclear masses. Nuclei with mass numbers larger than or equal to 50 have been selected for the investigation. These are 1919 in number (including even - even, even - odd, odd - even as well as odd - odd).

Results and Discussion

The present calculations proceed in two steps. Step 1: Liquid Drop Model: For a system of \( N \) neutrons and \( Z \) protons, the liquid drop part of the total binding energy is assumed to be [6, 7]:

\[
E_{LDM} = a_v \left[ 1 + \frac{4k_v}{A^2} T_z (T_z + 1) \right] A + a_s \left[ 1 + \frac{4k_s}{A^2} T_z (T_z + 1) \right] A^{2/3} + \frac{3e^2 Z^2}{5r_o A^{1/3}} - C_4 \frac{Z^2}{A} + E_p
\]

where, \( A = N + Z; \) \( e^2 \) is square of electronic charge; \( T_z \) is third component of isospin, defined by \( \frac{N - Z}{2}; \) \( a_v, a_s, k_v, k_s, r_o \) and \( C_4 \) are parameters of the model. \( E_p \) is the pairing energy, and following Möller - Nix [4], it is defined by:

\[
E_p = \Delta_p , Z \text{ odd and } N \text{ even} = \Delta_n , N \text{ odd and } Z \text{ even} = \Delta_p + \Delta_n + \Delta_{np} , Z \text{ odd and } N \text{ odd} = 0 , N \text{ even and } Z \text{ even}
\]

with,

\[
\Delta_p = \frac{d_p}{Z^{1/3}} \quad \Delta_n = \frac{d_n}{N^{1/3}} \quad \Delta_{np} = \frac{d_{np}}{A^{2/3}}
\]

In these equations, \( d_p, d_n \) and \( d_{np} \) are parameters.

The experimental binding energies have been adopted from Audi’s 2003 evaluation [8], and the binding energies have been corrected for binding energy of electrons. Chi square fit to determine the free parameter yields \( \text{rms} \) of 2.47 MeV. The values of parameters obtained are: \( a_v = -15.5810; k_v = -1.8199; a_s = 18.4576; k_s = -2.3075; r_o = 1.1975; \) \( C_4 = 1.7891; d_n = 5.1857; d_p = 5.0536; d_{np} = -8.7129. \)

Step 2: The fluctuations: Treatment of fluctuations is the most difficult part of the exercise. It is clear that the fluctuations include shell energy as well as microscopic pairing energy, and both of these depend on shell structure. There could in principle be a number of ways to parameterise the fluctuations, but the simplest way to do that is through a discrete Fourier transform inspired ansatz for the fluctuations. Assume for the sake of sim-
plicity that the fluctuations depend on number of neutrons outside a ‘core’ and number of protons outside the suitable ‘core’. Thus, we here have the case of a two dimensional discrete Fourier transform, with variables given by:

\[ x_1 = \frac{|N - N_c|}{N_c}, \quad x_2 = \frac{|Z - Z_c|}{Z_c} \]  

The values of \( N_c \) are assumed to be 2, 8, 20, 50, 82, 126 and 172 whereas the values of \( Z_c \) are assumed to be 2, 8, 20, 50, 82, 126. The ‘core’ for a given neutron number is obtained by looking at its proximity to adjacent magic numbers. For example, for \( N = 16 \), \( N_c \) is assumed to be 20.

The fluctuations, defined by:

\[ \delta E = E_{\text{Expt}} - E_{\text{LDM}} \]  

are parameterised as:

\[
\begin{align*}
\delta E &= \sum_{n=1}^{N} \sum_{m=1}^{M} a_{nm} \cos \left( 2\pi \frac{nx_1}{N} + 2\pi \frac{mx_2}{M} \right) \\
&\quad + \sum_{n=1}^{N} \sum_{m=1}^{M} b_{nm} \sin \left( 2\pi \frac{nx_1}{N} + 2\pi \frac{mx_2}{M} \right)
\end{align*}
\]  

with \( a_{nm} \) and \( b_{nm} \) being free parameters of the model. In practice, \( N \) and \( M \) are taken to be equal, hence, for a given \( N \), this model has \( 2N^2 \) parameters. The value of \( N \) is assumed to be 3 here. Chi square minimisation for these parameters yields rms deviation of 1.15 MeV, which is a tremendous improvement over the LDM binding energies. The resulting difference between the experimental and corresponding binding energies are plotted in Fig. (1) (denoted by ‘LDM+FT’). It is very clearly seen that the strong shell effects are well taken care of, the resulting difference is much smoother. This difference is required to be parameterised further. Work along these lines is in progress.

Recently, Bhagwat [10] has been able to reduce the rms deviation to merely 266 keV, which is the smallest reported in the literature so far, using a mic - mac model inspired by the semi - classical trace formulas [9].

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References