Non-perturbative aspects of quark gluon plasma above deconfinement temperature

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The structure of QCD, at least near to transition temperature, seems to be more complex than that presented by a weakly interacting gas of quarks and gluons. Perturbative predictions are upset by the presence of strong non-perturbative effects from the infrared sector of the theory as evinced by T^2 scaling of the trace anomaly. These non-perturbative effects can be parameterized by non-vanishing local condensates of various mass dimension. Through the operator product expansion, these condensates appear with power corrections in physical observable which provide non-perturbative information in addition to perturbatively calculable radiative corrections. We discuss how operator product expansion can be applied to calculate nonperturbative propagators and vertices of QCD above but close to deconfinement temperature. Correlated applications of these techniques in the diagnostic of quark gluon plasma are also indicated.

1. Introduction

There are strong circumstantial evidence that ongoing relativistic heavy ion collision experiments have been successful in creating a hot deconfined phase of quantum chromodynamics (QCD) in the laboratory. Owing to the asymptotic freedom, QCD matter at high enough temperature or baryon density is expected to behave like a *quasifree gas* of quarks and gluons called quark gluon plasma (QGP) [1].

It came as a surprise when it was observed that hadrons emitted during the high energy nucleus-nucleus collision exhibit large amount of collective flow. Common wisdom is that the collective flow is mostly generated early in the nuclear collision before the partons fragment or coalesce into hadrons. The success of the ideal hydrodynamic simulation to reproduce flow data [2, 3] is a strong indication that matter created in such collision is strongly interacting. It thermalizes rapidly. It behaves like a *near perfect fluid*, *i.e*, has very small shear viscosity (η) to entropy density (s) ratio. This is different from a weakly interacting perturbative gas of quarks and gluons which has large mean free path and hence large viscosity.

The highest temperature of the QGP in heavy ion collision is not more that $4-5\,T_c$, where T_c is a *pseudo-critical* temperature characterizing the rearrangement of the spectrum due to deconfinement. The QGP will then expand and cool before being realized as hadrons and electromagnetic radiations. A quantity which is very sensitive to nonperturbative dynamics of QCD is normalized trace anomaly or interaction measure $\Delta = (\epsilon - 3p)/T^4$. The failure of most sophisticated perturbation theories to date [4] to reproduce the interaction measure computed nonperturbatively in lattice QCD clearly shows that the quark gluon plasma in this temperature region is highly non-perturbative.

It is widely accepted that the non-perturbative dynamics of QCD is signaled by the emergence of power corrections in physical observable. If the mass scale involved in the problem is much higher than QCD scale parameter Λ_{QCD}, then it is possible to introduce these non-perturbative corrections via local correlators of field operators such as quark condensate $\langle\bar{\psi}\psi\rangle$ or gluon condensate $\langle G^a_{\mu\nu}G^{a\mu\nu}\rangle$ [1]. The quark condensate is associated with the spontaneous breaking of chi-

[1] The condensates are normal ordered correlators and vanish in a perturbative vacuum at zero temperature. At finite temperature, ensemble average of

ral symmetry and the gluon condensate characterizes the breaking of scale invariance of the QCD Lagrangian by the presence of a current quark mass or through quantum corrections. This approach of operator product expansion (OPE) has met noticeable success in QCD sum rule calculations at zero as well as at finite temperature [5–7].

Lately operator product expansion have been extensively used to calculate QCD propagator and vertices at zero temperature using various available techniques [8–11]. Some progress have also been made in this direction at finite temperature as well. In-medium nonperturbative chiral quark propagator [12] and quark photon vertex [13] in presence of dimension four electric and magnetic condensates were investigated some time ago. Also attempts were made to extract non-perturbative electric screening mass from the gluon propagator [14]. With temperature dependent condensates taken from lattice QCD, operator product expansion can provide valuable insight about analytic structure of nonperturbative QCD Green functions above but close to T_c where the reliability of perturbation theory is at stake.

In the original representation of the operator product expansion, no dimension two condensate appear. However, there are growing evidences from lattice simulations as well as from phenomenology that lowest order BRST invariant gluon condensate is of mass dimension two [15–19]. Simplest choice for such an operator is $\left\langle A^2 \right\rangle$. This operator is not gauge invariant and that is the reason why it does not show up in QCD sum rules which deal only with gauge invariant correlators. However, when one considers a gauge variant object such as propagator, the issue of gauge non-invariance does not seem to be compelling and any condensate with appropriate quantum number can appear in the OPE [20]. One can, of course, construct a gauge invariant albeit non-local $\left\langle A^2 \right\rangle$ by averaging it over all possible gauge transformations. A nonlocal but gauge invariant operator does not make sense from the perspective of operator product expansion. Alternatively, one may evaluate the operator in a given gauge. In particular, it has been shown that the bulk averaged $\left\langle A^2 \right\rangle$ attains a minimum in Landau gauge. Lattice simulations show that $\left\langle A^2 \right\rangle$ condensate in Landau gauge at zero temperature is a large quantity [15, 21, 22].

In fact, the scenario of dynamical generation of a $\langle A_0^2 \rangle$ condensate at finite temperature has been around the corner for some time. Possible phenomenological implications of such a scenario have been discussed in the context of static screening of chromoelectric fields [23] (see also [24] and references therein), Polyakov loop correlator [25], trace anomaly [19], dilepton production rate [26] or physics of heavy quarks [27] in the quark gluon plasma. Nonzero values of electric $\left\langle A_0^2 \right\rangle_T$ and $\left\langle A_i^2 \right\rangle_T$ have been confirmed on lattice [28, 29] for $SU_c(2)$ [2]. So far no lattice measurements for these condensates have been performed for $SU_c(3)$ but it is not expected to be qualitatively very different.

The aim of the present article is to present a systematic framework based on operator product expansion to calculate the non-perturbative corrections to various QCD Green functions. The article is organized as follows. In Section 2, we introduce nonperturbative Green functions and relate condensates with the appropriate moments of the propagators and vertices. This is followed by a detailed exposition of quark, gluon and ghost propagators in Section 3 in nonperturbative background of gluon condensates. We do not consider chiral condensate here but it can be easily incorporated by the methods presented

normal ordered correlators do not vanish. At finite temperature, the nonperturbative condensates are defined by subtracting the perturbative part from the full ensemble average of the corresponding correlator. By the term condensate we shall always mean nonperturbative part of the corresponding composite operator in what follows.

[2] We use $\langle \cdots \rangle_T$ to denote ensemble average.

herein. We point out possible applications and summary of our results in Section 4.

2. Non perturbative Green functions and condensates

There are various ways to calculate the OPE coefficients diagrammatically. We shall adopt plane wave method [30] here. In this approach, The non-perturbative part of a Green function is defined as the difference between (unknown) full Green function and the perturbative part. The condensates are then identified with the various moments of nonperturbative Green function.

Let us consider gluonic operators first. In QCD, gluon polarization tensor is not transverse in general, $K^\mu \Pi_{\mu\nu}(K) \neq 0$. For an $O(3)$ invariant gauge fixing condition, the most general tensorial structure of the in-medium gluon self-energy can be written as (we omit trivial color factor) [31]

$$\begin{aligned} \pi_{\mu\nu}(\omega,k) &= \pi_l(\omega,k)\,\mathcal{P}^l_{\mu\nu} + \pi_t(\omega,k)\,\mathcal{P}^t_{\mu\nu} \\ &+ \pi_m(\omega,k)\,\mathcal{M}_{\mu\nu} + \tilde{\pi}(\omega,k)\,l_{\mu\nu}\,. \end{aligned} \tag{1}$$

Here, ω and k are the Lorentz invariant single particle energy and momentum respectively,

$$\begin{aligned} \omega &= u\cdot K\,, \\ k &= \left[(u\cdot K)^2 - K^2\right]^{\frac{1}{2}}\,, \end{aligned} \tag{2}$$

and u^μ is the four velocity of the heat bath. In the rest frame of the medium $u^\mu = \delta^\mu_0$. The projection operators are defined as [31],

$$\begin{aligned} \mathcal{P}^l_{\mu\nu} &= -\frac{1}{k^2K^2}\left(k^2u_\mu + \omega\widetilde{K_\mu}\right)\left(k^2u_\nu + \omega\widetilde{K_\nu}\right) \\ &= \frac{K^2}{\widetilde{K}^2}\bar{u}^\mu\bar{u}^\nu\,, \end{aligned} \tag{3a}$$

$$\mathcal{P}^t_{\mu\nu} = \eta_{\mu\nu} - u_\mu u_\nu - \frac{\widetilde{K_\mu}\widetilde{K_\nu}}{\widetilde{K}^2}\,, \tag{3b}$$

$$\mathcal{M}_{\mu\nu} = -\frac{1}{\sqrt{-2\widetilde{K}^2}}\left(\bar{u}_\mu K_\nu + \bar{u}_\nu K_\mu\right)\,, \tag{3c}$$

$$l_{\mu\nu} = \frac{K_\mu K_\nu}{K^2}\,. \tag{3d}$$

where $\widetilde{K_\mu} = K_\mu - \omega u_\mu$ and $\bar{u}^\mu = u^\mu - \frac{\omega}{K^2}K^\mu$. The projectors $\mathcal{P}^{\mu\nu}_l$ and $\mathcal{P}^{\mu\nu}_t$ are transverse with respect to K^μ and $\mathcal{M}^{\mu\nu}$ satisfies a weaker condition $K_\mu\mathcal{M}^{\mu\nu}K_\nu = 0$. The scalar structure functions in the self energy are extracted through appropriate projections,

$$\pi_l = \mathcal{P}^{\mu\nu}_l\pi_{\mu\nu}\,, \tag{4a}$$

$$\pi_t = \frac{1}{2}\mathcal{P}^{\mu\nu}_t\pi_{\mu\nu}\,, \tag{4b}$$

$$\pi_m = -\mathcal{M}^{\mu\nu}\pi_{\mu\nu}\,, \tag{4c}$$

$$\widetilde{\pi} = l^{\mu\nu}\pi_{\mu\nu}\,. \tag{4d}$$

Here, π_m and $\widetilde{\pi}$ measure the deviation from transversality. In high temperature perturbative QCD (pQCD), the transversality holds only in temporal axial gauge and Feynman gauge. However, departure from transversality is sub-leading in temperature so π_m and $\widetilde{\pi}$ can be neglected in pQCD. Such simplification is in general not possible when polarization tensor is expressed in terms of condensates.

Now from (1), the most general form of the gluon propagator $\mathcal{D}_{\mu\nu} = \mathcal{D}_{0,\mu\nu}\left(1+\pi_{\mu\nu}\mathcal{D}_{0,\mu\nu}\right)^{-1}$ can be written as,

$$\begin{aligned} \mathcal{D}_{\mu\nu} &= -\frac{\mathcal{P}^t_{\mu\nu}}{K^2-\pi_t} - \frac{2}{X_D}\left[X_l\mathcal{P}^l_{\mu\nu} + X_m\mathcal{M}_{\mu\nu}\right. \\ &+ \left.\widetilde{X}l_{\mu\nu}\right]\,, \end{aligned} \tag{5}$$

where,

$$X_D = 2\left(K^2-\pi_l\right)\left(\xi^{-1}K^2-\widetilde{\pi}\right)+\pi_m^2\,, \tag{6a}$$

$$X_l = \left(\xi^{-1}K^2-\widetilde{\pi}\right)\,, \tag{6b}$$

$$X_m = \pi_m\,, \tag{6c}$$

$$\widetilde{X} = \left(K^2-\pi_l\right)\,. \tag{6d}$$

Using the covariant gauge the Slavnov Taylor identity, $K^\mu\mathcal{D}_{\mu\nu}K^\nu = K^\mu\mathcal{D}_{0,\mu\nu}K^\nu = -\xi$, we get

$$\frac{2\left(K^2-\pi_l\right)K^2}{2\left(K^2-\pi_l\right)\left(\xi^{-1}K^2-\tilde{\pi}\right)+\pi_m^2} = \xi\,. \tag{7}$$

For $\xi \neq 0$. Eq. (7) can be written as,

$$\left(K^2-\pi_l\right)\tilde{\pi} = \frac{\pi_m^2}{2} \tag{8}$$

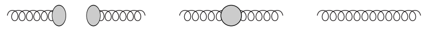

FIG. 1: Graphical Representation of (9). Full and nonperturbative propagators are denoted by circular and oval blobs respectively.

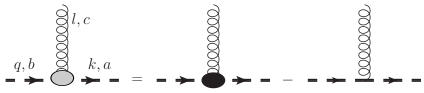

FIG. 2: The nonperturbative ghost-gluon vertex

This leaves three independent components in (5). So the most general form of the 'non perturbative' gluon propagator should be,

$$\begin{aligned} D_{\mu\nu} &\stackrel{\text{def}}{=} \mathcal{D}_{\mu\nu} - \mathcal{D}^{\text{pert}}_{\mu\nu} \\ &= P^l_{\mu\nu} D_l\left(\omega, k\right) + P^t_{\mu\nu} D_t\left(\omega, k\right) \\ &+ \mathcal{M}_{\mu\nu} D_m\left(\omega, k\right) , \end{aligned} \tag{9}$$

in an obvious notation. Note that, D_m is absent in covariant gauges if 1) $\xi = 0$ (Landau gauge) or 2) the self energy is transverse $\pi_m = \tilde{\pi} = 0$.

Gluon condensates of various dimension are related to the moments of the nonperturbative gluon propagator. In the rest frame of the medium ($u^\mu = \delta^\mu_0$), the condensates are given by,

$$\left\langle A_0^2 \right\rangle_T = -T\left(N_c^2 - 1\right) \int \frac{d^3k}{\left(2\pi\right)^3} D_l\left(0, k\right) , \tag{10a}$$

$$\left\langle A_i^2 \right\rangle_T = 2T\ \left(N_c^2 - 1\right) \int \frac{d^3k}{\left(2\pi\right)^3} D_t\left(0, k\right) , \tag{10b}$$

$$\left\langle \mathcal{E}^2 \right\rangle_T = -T\left(N_c^2 - 1\right) \int \frac{d^3k}{\left(2\pi\right)^{D-1}} D_l\left(0, k\right) k^2 \tag{10c}$$

$$\left\langle \mathcal{B}^2 \right\rangle_T = 2T\left(N_c^2 - 1\right) \int \frac{d^3k}{\left(2\pi\right)^{D-1}} D_t\left(0, k\right) k^2 . \tag{10d}$$

where N_c is the number of color. We have restricted here to the lowest Matsubara mode ($k_0 = 0$) in the frequency sum. This is equivalent to restricting to the most dominant infrared sector. Intuitively, the physical content of this approximation can be understood as follows : condensates reflect long distance property of QCD. Gluons with nonzero Matsubara mode propagate only short distance $\sim 1/T$ and can not feel the nonperturbative fluctuations. Also, it is obvious that this approximation is crucial to relate the condensates unambiguously to the moments of the gluon propagator.

Under renormalization, the gluon condensates in (10) acquire admixture of certain other operators vanishing on classical equations of motion or BRST variation of other operator [3]. As we will see later, this entails that in order to fix the Wilson coefficient of $D = 4$ gluon condensates in gluon polarization tensor, one need to calculate Wilson coefficients of ghost-antighost and mixed ghost-gluon condensates also.

The dimension four ghost-antighost condensate can be defined similar to gluon condensates,

$$\left\langle \bar{\eta} \Box \eta \right\rangle_T = T\left(N_c^2 - 1\right) \int \frac{d^3k}{\left(2\pi\right)^3} G\left(0, k\right) k^2 , \tag{11}$$

where G is the nonperturbative ghost propagator. The moment of the nonperturbative ghost-gluon vertex in Fig. 2 can be obtained as,

$$\begin{aligned} iT^2 \int \frac{d^3k}{\left(2\pi\right)^3} \frac{d^3l}{\left(2\pi\right)^3} g k_i \Gamma^{abc}_j\left(k, l\right) &= \frac{f^{abc}\delta_{ij}}{3N_c F_A} \\ &\times \left\langle g f^{lmn} \partial_\lambda \bar{\eta}^l A^{\lambda, m} \eta^n \right\rangle_T . \end{aligned} \tag{12}$$

3. Non perturbative QCD propagators

To calculate the nonperturbative corrections through OPE, the full Feynman diagrams to a given order in α_s are written down and equivalent perturbative ones are subtracted. The soft loop momenta are expanded in powers of external momenta and

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[3] $g^2 G^a_{\mu\nu} G^{a\mu\nu}$ is RG invariant.

moments are identified with the corresponding condensates from (10).

A. Quark self energy

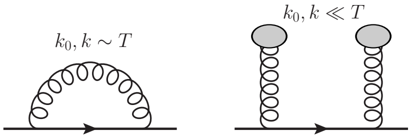

FIG. 3: One loop quark self energy. (Left) Hard thermal loop (HTL) correction (Right) condensate contribution to quark self energy.

To elucidate, let us start with the quark self energy. To one loop order, the perturbative and nonperturbative corrections are given by the diagrams in Fig. 3. The leading order thermal correction comes from the region in loop integration where the scale of momenta flowing through the gluon and quark lines are hard, *i.e*, $\sim T$. Now, let's assume that momentum of gluon line is very small $k_0 = 0, k \ll T$. In that case, gluon propagates a large distance and feels the nonperturbative fluctuations. If the intensity of this fluctuation (the condensates) are known then the rest of the sub-diagram can be calculated perturbatively because it contains only short distance physics.

Now, the general expression for in-medium fermion self energy in the chiral limit can be written as,

$$\Sigma\left(P\right) = -a\left(\omega,p\right) \not{P} - b\left(\omega,p\right) \not{u}\,, \qquad (13)$$

where the scalar functions are given by,

$$a\left(\omega,p\right) = \frac{1}{4p^2}\left[\mathrm{Tr}\left(\not{P}\Sigma\right) - \omega \mathrm{Tr}\left(\not{u}\Sigma\right)\right]\,, \qquad (14a)$$

$$b\left(\omega,p\right) = \frac{1}{4p^2}\left[P^2\mathrm{Tr}\left(\not{u}\Sigma\right) - \omega \mathrm{Tr}\left(\not{P}\Sigma\right)\right]\,. \qquad (14b)$$

Note that, a and b contain nonperturbative contribution from gluon condensates and perturbative contribution given by HTL corrections as,

$$a \;=\; a^{\langle A^2\rangle} + a^{\langle G^2\rangle} + \cdots + a^{\mathrm{HTL}}\,,$$

$$b \;=\; b^{\langle A^2\rangle} + b^{\langle G^2\rangle} + \cdots + b^{\mathrm{HTL}}\,. \qquad (15)$$

The leading order perturbative contribution is well known [32]. Contribution of $D = 4$ gluon condensate were calculated in [12] and $D = 2$ condensate in [33]

$$a^{\mathrm{HTL}} = -\frac{2\pi\alpha_s T^2}{3k^2} Q_1\left(\frac{\omega}{k}\right)\,, \qquad (16a)$$

$$a^{\langle A^2\rangle} = -\frac{2\pi\alpha_s}{3P^2}\left[\langle A_i^2\rangle_T - \langle A_0^2\rangle_T\right]\,, \qquad (16b)$$

$$a^{\langle G^2\rangle} = -\frac{2\pi\alpha_s}{3}\frac{1}{(\omega^2-k^2)^3}\left[\left(\frac{1}{3}k^2 - \frac{5}{3}\omega^2\right)\right.$$
$$\left.\times\,\langle E^2\rangle_T - \left(\frac{1}{5}k^2 - \omega^2\right)\langle B^2\rangle_T\right]\,, \qquad (16c)$$

$$b^{\mathrm{HTL}} = \frac{2\pi\alpha_s T^2}{3k}\left[\frac{\omega}{k}Q_1\left(\frac{\omega}{k}\right) - Q_0\left(\frac{\omega}{k}\right)\right)\,, \qquad (16d)$$

$$b^{\langle A^2\rangle} = -\frac{2\pi\alpha_s p_0}{3P^2}\left[2\langle A_0^2\rangle_T + \frac{2}{3}\langle A_i^2\rangle_T\right]\,, \qquad (16e)$$

$$b^{\langle G^2\rangle} = -\frac{16\pi\alpha_s}{9}\frac{\omega}{(\omega^2-k^2)^3}\left[\omega^2\langle E^2\rangle_T\right.$$
$$\left. + \frac{1}{5}k^2\langle B^2\rangle_T\right]\,. \qquad (16f)$$

Here Q_ns are Legendre function of second kind.

Then, the chiral quark propagator $S^{-1}(P) = \not{P} - \Sigma$ follows as

$$S\left(p_0,p\right) = \frac{\gamma_0 - \gamma\cdot\hat{\vec{p}}}{2D_+\left(p_0,p\right)} + \frac{\gamma_0 + \gamma\cdot\hat{\vec{p}}}{2D_-\left(p_0,p\right)}\,, \qquad (17)$$

where,

$$D_\pm\left(p_0,p\right) = \left(-p_0 \pm p\right)\left(1+a\right) - b\,. \qquad (18)$$

B. Gluon self Energy

The gluon self energy with gluon condensate follows from Fig. 4. Unlike quark self energy, we work here to the effective order in α_s in the sense that terms which are higher order in coupling constant are related to the terms of the order of α_s through the equations of motion. Various scalar structure functions in

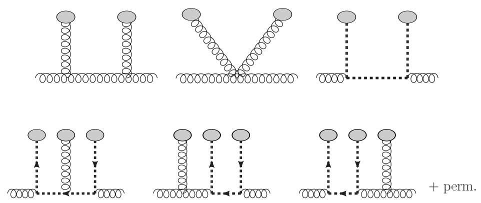

FIG. 4: Gluon self-energy with gluon condensate (1st and 2nd diagrams), ghost condensate (3rd diagram) and ghost-gluon mixed condensates (4th, 5th, 6th plus their permutation).

the self energy as given in (4) with $D = 2$ condensates are obtained as,

$$\pi_l^{\langle A^2\rangle} = f_i \frac{\alpha_s}{\pi} \langle A_i^2 \rangle_T - f_0 \frac{\alpha_s}{\pi} \langle A_0^2 \rangle_T \,, \tag{19a}$$

$$\pi_t^{\langle A^2\rangle} = g_i \frac{\alpha_s}{\pi} \langle A_i^2 \rangle_T - g_0 \frac{\alpha_s}{\pi} \langle A_0^2 \rangle_T \,, \tag{19b}$$

$$\pi_m^{\langle A^2\rangle} = \sqrt{2}\left(h_i \frac{\alpha_s}{\pi} \langle A_i^2 \rangle_T - h_0 \frac{\alpha_s}{\pi} \langle A_0^2 \rangle_T \right) , \tag{19c}$$

$$\widetilde{\pi}^{\langle A^2\rangle} = w_i \frac{\alpha_s}{\pi} \langle A_i^2 \rangle_T - w_0 \frac{\alpha_s}{\pi} \langle A_0^2 \rangle_T \,. \tag{19d}$$

with

$$f_0\left(p_0,p\right) = \frac{4\pi^2 N_c}{(N_c^2-1)} \left\{ \left(-\frac{\vec{p}^2}{P^2} + \frac{4p_0^2}{P^2} + \frac{p_0^2\vec{p}^2}{P^4} - \frac{4p_0^4}{P^4} - \frac{3p_0^4}{\vec{p}^2 P^2} + \frac{3p_0^6}{\vec{p}^2 P^4} \right) + \chi \left(\frac{\vec{p}^2}{P^2} - \frac{2p_0^2}{P^2} - \frac{2p_0^2\vec{p}^2}{P^4} \right.\right.$$
$$\left.\left. + \frac{4p_0^4}{P^4} + \frac{p_0^4}{\vec{p}^2 P^2} + \frac{p_0^4\vec{p}^2}{P^6} + \frac{2p_0^6}{P^6} - \frac{2p_0^6}{\vec{p}^2 P^4} + \frac{p_0^8}{\vec{p}^2 P^6} \right) \right\} , \tag{20a}$$

$$f_i\left(p_0,p\right) = \frac{2\pi^2 N_c}{(N_c^2-1)} \left\{ \left(-\frac{2\vec{p}^2}{P^2} + \frac{2p_0^2}{P^2} - \frac{4\vec{p}^4}{3P^4} + \frac{4p_0^2\vec{p}^2}{P^4} - \frac{8p_0^4}{3P^4} \right) + \chi \left(\frac{p_0^2\vec{p}^2}{P^4} + \frac{p_0^2\vec{p}^4}{3P^6} - \frac{2p_0^4}{P^4} \right.\right.$$
$$\left.\left. - \frac{5p_0^4\vec{p}^2}{3P^6} + \frac{7p_0^6}{3P^6} + \frac{p_0^6}{\vec{p}^2 P^4} - \frac{p_0^8}{\vec{p}^2 P^6} \right) \right\} , \tag{20b}$$

$$g_0\left(p_0,p\right) = \frac{4\pi^2 N_c}{(N_c^2-1)} \left\{ \left(-1 + \frac{11p_0^2}{2P^2} \right) + \chi \left(-\frac{p_0^2}{P^2} + \frac{p_0^4}{2P^4} - \frac{p_0^4}{2\vec{p}^2 P^2} - \frac{p_0^6}{2\vec{p}^2 P^4} \right) \right\} \tag{20c}$$

$$g_i\left(p_0,p\right) = \frac{2\pi^2 N_c}{(N_c^2-1)} \left\{ \left(\frac{5}{2} + \frac{\vec{p}^2}{P^2} - \frac{11p_0^2}{3P^2} \right) + \chi \left(\frac{\vec{p}^2}{6P^2} + \frac{p_0^2\vec{p}^2}{6P^4} + \frac{2p_0^4}{3P^4} - \frac{p_0^4}{2\vec{p}^2 P^4} + \frac{p_0^6}{2\vec{p}^2 P^4} \right) \right\} \tag{20d}$$

$$h_0\left(p_0,p\right) = \frac{4\pi^2 N_c}{(N_c^2-1)} \left\{ -\frac{p_0}{|\vec{p}|} + \frac{p_0^3}{|\vec{p}P^2|} \right\} , \tag{20e}$$

$$h_i\left(p_0,p\right) = \frac{2\pi^2 N_c}{(N_c^2-1)} \left\{ \frac{p_0}{|\vec{p}|} + \frac{p_0\,|\vec{p}|}{3P^2} - \frac{p_0^3}{|\vec{p}P^2|} \right\} , \tag{20f}$$

$$w_0\left(p_0,p\right) = 0\,, \tag{20g}$$

$$w_i\left(p_0,p\right) = 0. \tag{20h}$$

At zero temperature, $k^\mu \pi_{\mu\nu}^{\langle A^2\rangle} = 0$. We find that the transversality is weaker at finite temperature $k^\mu \pi_{\mu\nu}^{\langle A^2\rangle} k^\nu = 0$.

The $D = 4$ condensate contribution to the gluon self energy is also computed [34]. It is fairly lengthy but simplifying expressions can be obtained in certain limits. For example, static limit of the polarization tensor which gives electric and magnetic screening lengths

follow as [35],

$$\begin{aligned}\pi_l^{\langle G^2\rangle}(0,p) &= -\frac{a}{p^2}\,, \\ \pi_t^{\langle G^2\rangle}(0,p) &= -\frac{b}{p^2} - \frac{c}{p^2} + \frac{d}{p^2}\,. \qquad (21)\end{aligned}$$

Where,

$$a = \frac{4\pi^2 N_c}{F_A}\left[\frac{8}{3}\frac{\alpha_s}{\pi}\left\langle \mathcal{E}^2\right\rangle_T + \frac{8}{30}\frac{\alpha_s}{\pi}\left\langle \mathcal{B}^2\right\rangle_T\right], \qquad (22a)$$

$$b = \frac{4\pi^2 N_c}{F_A}\left[W_E\frac{\alpha_s}{\pi}\left\langle \mathcal{E}^2\right\rangle_T - W_B\frac{\alpha_s}{\pi}\left\langle \mathcal{B}^2\right\rangle_T\right], \qquad (22b)$$

$$c = R\xi\left[\langle\bar{\eta}^a\Box\eta^a\rangle_T - \left\langle g f^{abc}\partial_\mu\bar{\eta}^a A^{\mu,b}\eta^c\right\rangle_T\right], \qquad (22c)$$

$$d = RW_M\left[\langle\bar{\eta}^a\Box\eta^a\rangle_T - \left\langle \mathcal{B}^2\right\rangle_T + \left\langle \mathcal{E}^2\right\rangle_T + \cdot\cdot\right], \qquad (22d)$$

and $R = \frac{4\pi\alpha_s N_c}{3F_A}$, $W_E = \left(2+\frac{\xi}{3}\right)$, $W_B = \frac{1}{15}(38+9\xi)$ and $W_M = (2+\xi)$, $F_A = N_c^2 - 1$. The condensates which appear in last two terms of π_t are classical equations of motion and hence vanish. The value of a obtained here is same as in Ref. [14].

The screening masses are the given by,

$$\begin{aligned} m_D^2 &= \pi_l^{pert}\left(0,-m_D^2\right) + \pi_l^{np}\left(0,-m_D^2\right)\,, \qquad (23) \\ m_m^2 &= \pi_t^{pert}\left(0,-m_m^2\right) + \pi_t^{np}\left(0,-m_m^2\right)\,. \qquad (24)\end{aligned}$$

Here, m_D and m_m are electric and magnetic screening masses respectively. Nonperturbative contribution to self energy follow from (19) and (21), $\pi_l^{pert} = 4\pi\alpha_s T^2$ for pure gauge theory and $\pi_t^{pert} = 0$.

Let us note that, unlike case of quark, gluon self energy is explicitly gauge dependent rendering its physical interpretation to be doubtful.

C. Ghost self energy

Similarly, $D = 2$ gluon condensate contribution to ghost self energy in covariant gauge can be evaluated from Fig. 5 as,

$$\Pi(p_0,p) = \frac{4\pi\alpha_s N_c}{N_c^2-1}\left[\frac{p_0^2}{P^2}\left\langle A_0^2\right\rangle_T - \frac{1}{2}\left(\frac{4}{3} - \frac{p_0^2}{P^2}\right)\left\langle A_i^2\right\rangle_T\right]. \qquad (25)$$

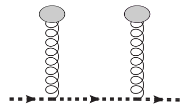

FIG. 5: Gluon condensate contribution to ghost self energy

There is no HTL correction for ghost, but ghost can receive non-perturbative mass corrections in a background characterized by non-vanishing condensates. One can similarly calculate Wilson coefficients of $D = 4$ gluon condensates to ghost self energy. To fix these coefficients uniquely, one has to go beyond one loop and calculate Wilson coefficients of $D = 4$ ghost-anti ghost $(\bar{\eta}\Box\eta)$, and mixed ghost-gluon condensate $\left(f^{abc}\partial_\mu\bar{\eta}^a A^{\mu,b}\eta^c\right)$. This is similar to gluon self energy calculation at one loop [35].

4. Outlook

We have provided a theoretical framework based on operator product expansion to calculate QCD Green functions in the deconfined phase. The crucial input to this formulation are the nonperturbative condensates. In the context QCD sum rule, the condensates are phenomenological parameters which are tuned to fit the experimental data. A promising method is to determine them from first principle in lattice QCD. Such calculations, at least in the vicinity of transition region, indeed exist and provide important clue about correlation of deconfinement of color degrees of freedom and melting of vacuum condensates [29, 36].

At high enough temperature, $T \gg T_c$ the quark gluon plasma is perturbative and condensates are expected to play minor role. However, close to T_c the QGP is highly nonperturbative and may even upset the perturbative behaviour. We have shown here how the propagation of quarks and gluons are modified in a background of gluon condensates. Such information can be used to a calculate a number of observables in a strongly coupled

quark gluon plasma, *e.g*, electromagnetic radiation, jet quenching, EOS etc. Work along this direction are currently under progress.

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