Band structures in nuclei with $Z, N$ close to the magic number 82

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The structure of the nuclei, with proton and/or neutron numbers close to the spherical magic numbers, depends largely on the shape driving effects of the valance orbitals. The high spin states of Thallium and Bismuth nuclei in $A = 190$ mass region and Cesium nuclei in $A = 130$ mass region have been studied to investigate the band structures in these nuclei. It has been observed that an onset of deformation sets in for the Bi isotopes, with $Z$ value just above the magic number 82 and at neutron number 112. The band structures in the Tl isotopes, with $Z$ just below the magic number 82, are dominated by the intruder configuration. On the other hand, a change in the structure of the band, built on the unique parity $\pi h_{9/2} \otimes v h_{11/2}$ configuration in the Cs isotopes, has been observed at neutron number $N = 79$ while approaching the neutron magic number. The principal axis cranking calculations have been performed for these nuclei to understand the different band structures in these nuclei.

1. Introduction

The single particle orbitals near the magic numbers can be investigated from the gamma ray spectroscopic studies of the odd-A nuclei with proton and/or neutron numbers close to the magic number. For example, the ground state ($J^\pi = 9/2^-$) of $^{209}$Bi ($Z = 83, N = 126$) corresponds to the odd-proton in the $h_{9/2}$ orbital situated just above the magic number at $Z = 82$. The excited states in this nucleus are due to the excitation of the odd proton in different orbitals above the $h_{9/2}$. Indeed, the observed $7/2^-, 13/2^+, 5/2^-, 3/2^-$ states in $^{209}$Bi have been identified as the spherical shell model states of $f_{7/2}, i_{13/2}, f_{5/2}$ and $p_{3/2}$, respectively [1]. The relative positions of these orbitals, which are important to test the model predictions, can be inferred from the excitation energies of these states. The $^{209}$Bi, with a doubly magic core, is a perfect example of a spherical shell model nucleus. For a spherical nucleus, the angular momenta are generated by single particle (or hole) excitations. The collective modes, in these nuclei, are realized in the form of vibrational states. The ratios of the excitation energies of $E(I)/E(I - 2)$ ($I = 4, 6$ or 8) give the information about the method of generation of angular momentum $I$ in a nucleus. The values of these ratios are shown in Fig. 1 for the polonium ($Z = 84$) isotopes as a function of neutron number $N$. The limits of these ratios for the vibrational and the two quasi-particle excitation in spherical $h_{9/2}$ orbital are also shown. The $E(4)/E(2)$ ratio is much lower than the collective vibrational limit for the $N = 126$ shell closure. All the three ratios seem to deviate from the spherical vibrational or two proton excitation limits and thereby indicating a structural change for neutron numbers below 114.

![Fig. 1: Systematic of the ratio of $E(I)/E(I-2)$ for Po isotopes (see text for details).](image-url)

Rotational band structures, a characteristic of a deformed nucleus, have indeed been observed for the very neutron deficient isotopes of $^{191,193}$Bi [2]. On the other hand, no indication of such rotational band structure observed in $^{197}$Bi. A magnetic rotational band has been observed in this nucleus at the excitation energy of $> 4$ MeV suggesting a small deformation at high excitation in this nucleus. Therefore, in other words, the neutron magic gap at $N = 126$ seems to reinforce the $Z = 82$ magic gap until at least $N = 114$ to induce spherical shapes in the heavy mass Bismuth nuclei. Thus it is interesting to study whether this reinforcement continues up
to even lower values of the neutron number or breaks down to indicate an onset of deformation in the Bi isotopes at N = 112. But the information on the excited states in $^{195}$Bi was very limited to address this question. The shape driving Nilsson orbitals $\pi[505]9/2^-$ and $\pi[660]13/2^+$ induces oblate shape in the Tl and Bi nuclei in the $A = 190$ mass region. Both the above orbitals are the intruder proton orbitals for the Thallium ($Z = 81$) nuclei. These Nilsson orbitals, originated from the $\pi h_{9/2}$ and $\pi_{13/2}$ states, respectively, intrudes from the major shell above $Z = 82$ in to the shell below it for oblate deformation. These orbitals, therefore, play a significant role in breaking the spherical symmetry in a nucleus close to spherical magic number. Hence, it is important to identify these intruder orbitals and to study the band structure built on these configurations in Tl nuclei.

Moreover, the proton Fermi levels, for the nuclei in this region, lie in the lower part where as, the neutron Fermi levels lie in the upper part of the shell between the magic numbers 82 and 126. This presents an ideal situation for exotic phenomena of collective and single particle excitations, like magnetic rotation (MR) and chirality [3, 4] to occur in these nuclei. These resulted in the observation of shears band and the chiral doublet bands, respectively, in a nucleus.

On the other hand, for the nuclei in mass region $A = 130$, the proton and the neutron Fermi levels lie, respectively, in the lower and upper halves of the shell between the magic numbers 50 and 82. Therefore, one would expect similar single particle and collective bands to occur in the nuclei in this region. MR bands have been reported in many of the nuclei in this region [5 - 7]. One of the interesting aspects in the nuclei in this region is that both the protons and the neutrons occupy the $\pi h_{11/2}$ orbital. However, as they are in the lower and upper part of this orbitals, the respective Nilsson components have opposite (towards prolate and oblate, respectively) shape driving effects. This would lead to $\gamma$-softness and triaxiality in the nuclei in this region. This is reflected in the calculated total Routhian surfaces in the nuclei in this region [8]. In this study, the total Routhian surface energies were calculated for the N = 79 isotones of Pr, La, Cs etc. nuclei in the $\pi h_{11/2}$ configuration. It was found that not only the stable triaxiality and $\gamma$-softness dominate this region but also the shapes change quite rapidly with particle number and rotational frequency. The band structure, for the same configuration, depends on the shape of a nucleus. For example, the chiral doublet band structure can be obtained in a nucleus with stable triaxiality. The $\gamma$-softness destroys the chiral symmetry. The observation of Chiral partner bands have been reported in the odd-odd Cesium ($Z = 55$) isotopes with neutron number N = 71 to N = 77 [9 - 11], for the $\pi h_{11/2}$ $\otimes$ $\nu h_{11/2}$ configuration. However, as the neutron number increases toward the N = 82 magic gap, it needs to be investigated if the similar band structure persist for the heavier isotopes as well. The shape driving effects of the neutrons, close to the spherical shell closure N = 82, may not be strong enough to induce a stable triaxial shape and, there by, the potential energy surfaces tend to be $\gamma$ soft with small deformation which are ideal conditions for destroying the chiral arrangement and for the emergence of the MR bands. $^{132}$Cs is the heaviest isotope of Cs for which the $\pi h_{11/2}$ $\otimes$ $\nu h_{11/2}$ configuration was identified, with a rotational band structure along with a possible chiral partner [9]. The above configuration has not yet been identified in $^{134}$Cs. We have investigated the high spin states in $^{134}$Cs to search for the above configuration and to study the structure of the band built on this configuration.

Therefore, with the above motivation in mind, the high spin states in the odd-Z nuclei $^{195,198}$Bi and $^{194,197}$Tl in A = 190 region with proton number close to the Z = 82 magic gap and in the odd-odd $^{132}$Cs nucleus in A = 130 region with neutron number close to the magic gap N = 82 have been explored with the help of several collaborators in India. It is to be noted that both the odd-odd and odd-even isotopes of Bi and Tl were studied.

2. Experimental Techniques

The high spin states in the above nuclei were populated by fusion evaporation reactions using: (i) heavy ion beams from the K = 130 cyclotron at VECC, Kolkata, the 15-UD Pelletron at IUAC, New Delhi and the Pelletron Linac Facility at TIFR, Mumbai and (ii) light ion
(α) beam from VECC. The α-beam was important and useful to populate the intruder levels. The INGA array, consisted of clover HPGe detectors (8 – 15 in number, depending on the experiment), was used to detect the gamma rays. In the VECC experiment using α beam, a modest in-house facility, with a clover HPGe, a LEPS HPGe and a large volume (50% relative efficiency) single HPGe detector, was used along with a multiplicity filter array consisted of 50 BaF₂ detectors (25 each from top and bottom). The Fig. 2 shows the photograph of the above set up. The LEPS detector used in this experiment helped us to identify the low energy gamma rays more cleanly. The resolution (FWHM) of the LEPS detector was 1.2 keV compared to 1.8 keV in the clover for a 356-keV gamma ray.

The level schemes of the above nuclei were constructed from the coincidence correlation among the detected gamma rays. The data were sorted in to both Eγ-Eγ matrix and Eγ-Eγ-Eγ cube, wherever possible, to obtain the coincidence relations. One of the important criteria in the present study was to identify different proton and neutron configurations and the band structure build on those. It was important, therefore, to assign the definite spin and parity (Jπ) of the excited states to identify the configuration correctly. Therefore, much importance was given to determine the Jπ of the levels. One needs to know both the multipolarity (l) and the type (E/M) of the gamma ray de-exciting a level to determine its Jπ. These were deduced from the DCO ratio and the Polarization measurements. The geometry of the clover detectors allows one to deduce the integrated polarization ratio to determine the type of the gamma rays. The positive and negative value of this ratio indicates, respectively, electric (E) and magnetic (M) type of the gamma ray transition. The details of the experimental set up used in all the above experiments and the analysis techniques can be found in Ref. [12 – 15].

The polarization method was validated from the known type of gamma rays. Example of typical spectra for parallel and perpendicular scattering of a known electric and a known magnetic transition in 197Tl has been shown in Fig. 3. It can be clearly seen that the intensity of the perpendicular (parallel) scattered component is more than the parallel (perpendicular) one for the 695.4-keV (387.3-keV) gamma ray which is known to be of electric (magnetic) type.

3. Results

The results obtained for the nuclei studied in this work are the following:

3.1. 195Bi

The high spin states in 195Bi were studied using two reactions 181Ta(²⁶Ne,6n)195Bi at 134 MeV of beam energy obtained from the K=130
cyotron at VECC. The prompt level scheme in $^{195}$Bi was known up to 2046 keV of excitation energy including a 32 ns isomer at 13/2$^+$. Two more isomers with half-lives 750 ns (29/2$^-$) and 80 ns (25/2$^+$) were also observed [16]. However, the excitation energy of the 750 ns isomers was not known. No rotational band was reported in this nucleus in the previous study. In the present work, the prompt level scheme has been extended up to about 3 MeV and the excitation energy of the 29/2$^-$ isomer has been deduced to be 2395 keV from the observation of a 86-keV gamma ray de-exciting the above isomer. More importantly, the type of the 345-, 392- and 421-keV gamma rays have been found to be M1(+E2) (see Fig. 4) contrary to the E1 (first two) and E2 type reported in Ref. [16]. These assignments along with the five new transitions, suggest that a rotational band is built on the 13/2$^+$ isomer.

3.2. $^{198}$Bi

The level scheme of this odd-odd Bismuth isotope was known up to ~ 4 MeV [17]. Zwartz et al. [18] have tentatively assigned 3 MR bands to $^{198}$Bi without any observed connecting transition with its low lying states. In our study, using the $^{nat}$Re($^{16}$O,5n/7n)$^{195}$Bi reaction at the beam energy of 112.5 MeV, obtained from the 15UD pelletron at IUAC, New Delhi, we could establish the connection of these bands to the lower lying states and hence the excitation energy and configuration could be assigned to these MR bands. Therefore, the level scheme of $^{198}$Bi could be extended up to about 6.5 MeV.

3.3. $^{194}$Tl

The high spin states in $^{194}$Tl have been studied by Kreiner et al. [19] and have observed a band like structure built on a 8$^+$ state but the excitation energy and the J$^\pi$ of the states above it were tentative. In the present work, done at TIFR, Mumbai using $^{nat}$Re($^{13}$C, xn) reaction at 75 MeV of beam energy, the level scheme has been extended with 19 new $\gamma$-ray transitions and has been firmly established with definite excitation energy and J$^\pi$ assignment of the states. Two new band structures have been observed for the first time in this nucleus including a MR band built on a 6-quasiparticle band head [13].

3.4. $^{197}$Tl

The gamma ray spectroscopy of $^{197}$Tl has been studied at VECC, Kolkata using 48 MeV $\alpha$-beam on a $^{197}$Au target and using the detection facility shown in Fig. 1. A comparison of $\gamma$-ray spectrum obtained in LEPS and clover detectors have been shown in Fig. 5. In this study, the J$^\pi$ of the states could be firmly assigned from the DCO and IPDCO measurements. The parity of the 13/2 state at 1553 keV and 15/2 state at 2114 keV have been found to be positive as opposed to the negative ones tentatively assigned in the previous work [20]. So, the configurations of the bands could be established.
3.5. $^{134}$Cs

A total of 32 new transitions have been identified in this nucleus and were placed in a new and improved level scheme compared to the previous one [10]. Apart from several new positive and negative parity states, a band like structure, built on the 9$^+$ band head has been established for the first time in this odd-odd nucleus [12]. The experiment was performed at TIFR, Mumbai using two reactions: (i) $^{130}$Te($^7$Li,3n)$^{134}$Cs at 30 MeV and (ii) $^{130}$Te($^{11}$Li,$\alpha$3n)$^{134}$Cs at 52 MeV. A single and a sum of double gated spectra, obtained in this work are shown in Fig. 6.

![Fig. 6: (a) Single and (b) sum of double gated spectra of $^{134}$Cs from the present work.](image)

4. Theoretical Calculations and Discussion

The results obtained for the above nuclei have been discussed in the framework of cranking calculations [21, 22]. In these calculations, the total Routhian surfaces (TRSs) were obtained using a deformed Woods-Saxon potential and pairing interaction with the Strutinsky shell corrections method in the ($\beta_2,\gamma$) deformation mesh points and minimized with respect to the hexadecouple deformation ($\beta_4$). These were calculated at different rotational frequencies and for different configurations.

The calculated TRS for $^{195}$Bi in the $i_{13/2}$ configuration has been shown in Fig. 7. In these plots, $\gamma = 0^\circ$ ($\gamma = -60^\circ$) corresponds to prolate (oblate) deformation. A minimum is clearly seen in Fig. 7 at $\gamma = -64^\circ$ and $\beta_2 = 0.13$, indicating an oblate deformation in $^{195}$Bi. This is similar to the lighter isotopes $^{190,192}$Bi [2]. The rotational like band, based on the 13/2$^+$ isomer, observed in our experiment is commensurate with this calculations. In $^{191,193}$Bi, rotational bands have also been observed which were based on 11/2$^-$ state, originated from the coupling of $h_{9/2}$ orbital with the 2$^+$ of the even-even Pb core. An oblate deformation is predicted in the TRS calculations for the 11/2$^-$ configuration in $^{195}$Bi similar to $^{191,193}$Bi. This shows that an onset of deformation occur at $N = 112$ for the Bi isotopes when neutron number is decreased from the closed shell $N = 126$ and is in agreement with the Po isotopes shown in Fig. 1.

![Fig. 7: TRS plots for the +ve parity, +ve signature ($\pi_{i_{13/2}}$) configuration in $^{195}$Bi.](image)

For the Tl isotopes, $\pi_{h_{9/2}}$ and $\pi_{i_{13/2}}$ are the intruder orbitals and the 9/2$^+$[505] and 13/2$^+$[606] Nilsson orbitals, originated from the $\pi_{h_{9/2}}$ and $\pi_{i_{13/2}}$ become accessible for the odd-proton at low to moderate excitation energy for oblate deformation. Band structures based on these orbitals have been observed in light mass odd-$A$ Tl isotopes [23, 24] for which the neutrons, with Fermi level near the mid-shell, reinforce the oblate deformation. In $^{197}$Tl, we have identified the $i_{13/2}$ state at 1553 keV of excitation energy. A rotational band based on 9/2$^+$ band head has also been observed in this nucleus. These indicate a deformed structure even at $N = 116$ for the Tl nucleus with proton number close to the spherical magic gap at 82.

The valence neutrons occupy the $\nu_{i_{13/2}}$ orbital for the Tl isotopes in $A = 190$ region.
Therefore, a band structure based on $\pi h_{9/2} \otimes \nu i_{13/2}$ configuration is expected at low excitation for the odd-odd Tl isotopes. But, the band crossing phenomena has not been studied extensively in these nuclei. Both the first proton and the first neutron crossing are “Pauli blocked” in an odd-odd nucleus. Therefore, the first band crossing is observed at a higher frequency in these nuclei. Hence, unless the level scheme is not known up to several high spin states, the band crossing can not be studied in an odd-odd nucleus. The new and improved level scheme of $^{194}$Tl (up to $18 \hbar$ for this band), obtained in the present work, allowed us to study the band crossing and particle alignment in this nucleus. The crossing frequency of $\hbar \omega_0 = 0.34$ MeV in $^{194}$Tl, agrees well with the second crossing at $\hbar \omega_{2,2} = 0.36$ MeV in the neighboring even-even “core nucleus” $^{192}$Hg [25]. Therefore, drawing the similarity, the band crossing in $^{194}$Tl has been attributed to the alignment of a pair of neutrons.

The TRS calculations, a near oblate shape is obtained for the $\pi h_{9/2} \otimes \nu i_{13/2}$ configuration in $^{194}$Tl with $\beta_2 = 0.15$ and $\gamma = -57^\circ$, as shown in the top panel of Fig. 8. The quasi-particle Routhians, calculated using the above shape parameters, show two pairs of neutron alignments before the first proton pair alignment in $^{194}$Tl which is in agreement with the above conjecture.

The TRS calculations for the six quasi-particle configuration, corresponding to one of the side bands in $^{194}$Tl have been shown in the bottom panel of Fig. 8. It has a minimum at $\beta_2 = 0.06$ and $\gamma = -80^\circ$, indicating a near spherical shape for this configuration. The gamma rays, observed in this band, are found to be M1 in nature. The excited states in this band satisfy the criteria of an MR band [26]. A semi classical analysis, proposed by Macchiavelli et al. [27], has been performed for this band and a value of the interaction strength per particle-hole pair has been obtained as 332 keV, in good agreement with those obtained for the well known MR bands in the Pb nuclei in this region [28].

The shell model calculations have been performed [29] to understand the excited states in $^{134}$Cs using $^{100}$Sn as core and SN100PN interaction [30]. The low lying states could be explained well in these calculations. It has been found that the agreement improves when the valance protons are allowed in $g_{7/2}$, $d_{5/2}$, $s_{1/2}$ and $d_{3/2}$ orbitals. Also, it was crucial to include the $\pi h_{11/2}$ orbital for negative and high spin positive parity states.

The structure of the band built on the $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration in Cs isotopes.

The structure of the band built on the $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration in $^{134}$Cs, obtained in our work, looks very different than the same band observed for the lighter isotopes as shown in Fig. 9. Well developed rotational bands have been observed for the lighter isotopes. This band along with a side band with same $J^\pi$ value was interpreted as chiral partner band in $^{132}$Cs [9]. The gamma ray transitions in this band in $^{134}$Cs.
are mostly M1 with no cross over E2 transition and it does not have a well developed rotational band structure of a deformed nucleus. The excited states in this band seem to follow the criteria of MR bands. Clearly, a change in the structure of the $\pi h_{11/2} \otimes v h_{11/2}$ band is seen for the $N = 79$, neutron rich isotope of $^{134}$Cs [12].

The TRS calculations performed for the above configuration of Cs isotopes for $N = 71$ to 81 reveal that the shape changes from a stable triaxial deformation with $\gamma$ close to $-30^\circ$ for $N \leq 77$ to a shape with large amount of $\gamma$-softness for $N \geq 79$. This can be seen in Fig 10 in which the calculated values of the TRS energies are plotted as a function of the deformation parameter $\gamma$. These are plotted for the $\beta_2$ values close to the minimum in each case. An offset has been added with each plot in Fig. 10 for clarity. The calculated $\beta_2$ values obtained from the minimum of the TRS are shown in Fig. 11 for the above isotopes. It shows that the $\beta_2$ values decrease monotonically with mass number and it goes to less than 0.1 for $A \geq 134$.

Fig. 10: The TRS energies as a function of $\gamma$ deformation for Cs isotopes with $N = 71 - 81$.

The large gamma softness and very small deformation (below 0.1) are the ideal conditions for destroying the chiral symmetry and the emergence of magnetic rotation in a nucleus. The tilted axis cranking (TAC) calculations, performed for $^{132}$Cs and $^{134}$Cs indicate that the calculated B(M1) values of the excited states in the above band in $^{134}$Cs decrease with spin, a characteristic of a MR band. However, no such behavior has been observed for the calculated B(M1) values for lower lying states of the same band in $^{132}$Cs [12]. Therefore, it may be concluded that there is a transition from chiral symmetry breaking to magnetic rotation around $N = 79$ in the Cs isotopes for the $\pi h_{11/2} \otimes v h_{11/2}$ configuration. However, lifetime measurements of the states are necessary to observe the characteristic behavior of the transition strengths and there by to confirm the Chiral and MR nature of the bands in $^{132}$Cs and $^{134}$Cs.

Fig. 11: The calculated $\beta_2$ values obtained from the TRS for Cs isotopes with $N = 71 - 81$.

5. Summary

A variety of band structures has been observed for the Bismuth and Thallium isotopes in mass region $A = 190$ with proton number on either side of the spherical magic number $Z = 82$. For the Bi isotopes there seem to be an onset of deformation at $N = 112$ from the observation of rotational band based on $i_{13/2}$ state, in agreement with the predictions of the TRS calculations. In $^{198}$Bi, three previously observed MR bands have been connected to the lower states and thereby, the excitation energy and configurations of these bands could be established. In $^{197}$Tl, the intruder $\pi i_{13/2}$ state could be identified for the first time. In the odd-odd $^{194}$Tl, the $\pi h_{9/2} \otimes v i_{13/2}$ band has been extended beyond the band crossing and it has been properly characterized and discussed in the frame work of TRS calculations. Two other side bands in this nucleus have been identified for the first time and one of them, a six quasiparticle band, seems to be an MR band. A semi-classical approach has been adopted to discuss this MR band and particle-hole interaction strength has been deduced. In $^{134}$Cs, having neutron number close to $N = 82$, the $\pi h_{11/2} \otimes v h_{11/2}$ band has been identified for the
first time. This band has the characteristic of a MR band in sharp contrast to the bands built on the same configuration observed in the lighter odd-odd isotopes of Cs which were interpreted as chiral bands. The TAC calculations predict such transition in the nature of the band and it can be understood from the TRS calculations that a change in the potential energy surfaces is responsible for such a transition.

Acknowledgements

The help of all the collaborators of the above works, from different institutions and universities in India are gratefully acknowledged. Most part of the above work is the thesis work of Mr. H. Pai and his effort is acknowledged. The effort of the operators of the accelerators at VECC, TIFR and IUAC is acknowledged. I would also like to thank S. Kumar for the TAC calculations and P.C. Srivastava for the shell model calculations.

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