Double heavy quarkonium production in electron-positron annihilation at energy $\sqrt{s} = 10.6$ GeV

Elias Mengesha and Shashank Bhatnagar

Department of Physics, Addis Ababa University, P.O.Box 1176, Addis Ababa, Ethiopia.

In this work we study the exclusive production process $e^- + e^+ \rightarrow J/\psi + \eta_c$ at energy $\sqrt{s} = 10.6$ GeV observed at B-factories whose measurements have recently been done by Babar and Belle groups [1]. It is well known that there was a significant discrepancy between the experimental measurements [1] and the non-relativistic QCD (NRQCD) predictions for this process at centre of mass energies $\sqrt{s} \approx 10.6$ GeV. This process has been studied in a Bethe-Salpeter formalism under Covariant Instantaneous Approximation (CIA) [2,3] which is a Lorentz-invariant generalization of Instantaneous Approximation (IA).

We now calculate the cross section for the process $e^- + e^+ \rightarrow J/\psi + \eta_c$ for which our results are comparable with the data[1]. There are four Feynman diagrams in the leading order (LO) of QCD for the process $e^- + e^+ \rightarrow J/\psi$ meson. One of these is depicted in Fig.1. The other three diagrams can be obtained by permutations. The adjoint 4D wave functions for $\eta_c$ meson is $\Psi(P_v,q_b) = S_F(-q_2)\Gamma^V(\hat{q}_b)S_F(q_4)$ and for $J/\psi$ meson is $\Psi(P_v,q_a) = S_F(-q_1)\Gamma^\gamma(\hat{q}_a)S_F(q_1')$ where $q_a, q_b$ are the internal momenta of the hadrons $J/\psi$ and $\eta_c$ respectively with the corresponding hadron-quark vertex functions $\Gamma^\gamma$ and $\Gamma^\nu$ [2,3,4,5].

The details of momentum labeling of one of the Feynman diagrams contributing to this process is shown in Fig. 1 below. Using Feynman rules, one can obtain the amplitude for each of the diagrams. The amplitude corresponding to process in Fig.1 is given by

$$M_1 = \frac{e_d\epsilon_{ab}}{\sqrt{s}}(p_2)\gamma_\mu u(p_1) \int d^4q_0d^4q_b \times \text{Tr}[S_F(-q_2)\Gamma^V(\hat{q}_b)S_F(q_4)\gamma_\beta S_F(q_1)\gamma_\mu S_F(-q_2)\Gamma^\gamma(\hat{q}_a)S_F(q_1')\gamma_\alpha \frac{\delta_{ab}}{K^2}] \quad (1)$$

where $c = \frac{4}{3}$ is the color factor, the Mandelstam variable $s$ is defined as, $s = -(p_1 + p_2)^2$ and $e_Q = 2e/3$ is the electric charge of the charmed quark. In the heavy quark limit, total hadron momentum, $P \sim M$ and internal momentum $q << M$. In this limit, we can express the amplitude $M_1$ as

$$M_1 = -\frac{24\pi^2\alpha_{em}\alpha_v m_c^3}{s^2}e_Q\epsilon_{\alpha\beta\lambda\sigma}e_{\alpha\beta\lambda\sigma}P_b P_a \sigma[p(p_2)\gamma_\mu u(p_1)\xi_a \xi_b]. \quad (2)$$

Here $N_P$ and $N_c$ in $\xi_{a,b}$ (see Ref.[2] for details) are the BS normalizers for $\eta_c$ and $J/\psi$ respectively which are evaluated by using the current conservation condition. The values of BS normalizers thus obtained for $J/\psi$ and $\eta_c$ mesons are $N_c = 0.504 GeV^{-3}$ and $N_P = 0.410 GeV^{-3}$ respectively. The total amplitude for the process $e^-e^+ \rightarrow J/\Psi \eta_c$ can be obtained by summing over the amplitudes of all the four diagrams contributing to this process. The unpolarized total cross section is obtained by summing over various $J/\Psi$ spin-states and averaging over those of the initial state $e^-e^+$. Thus, in the CM frame the total cross section, $\sigma$, is given by [2]
\[ \sigma = \frac{2^{30} \pi^3 \alpha^2_{em} \alpha^2_s m_c^6}{83^3 s^4} \left( 1 - \frac{16 m_c^2}{s} \right)^2 \xi_s^2 \xi_b^2. \] (3)

In this paper we have calculated the cross section of the exclusive process of
\[ e^+ e^- \rightarrow J/\Psi \eta_c \] at energy \( \sqrt{s} = 10.6 GeV \)
in the framework of BSE under CIA [2] using only the leading order (LO) diagrams
in QCD. We find the theoretical value of
\[ \sigma[e^+ e^- \rightarrow J/\Psi \eta_c] = 21.75 fb \] (for details see [2]), which is broadly in agreement with
the Babar’s data \( \sigma[e^+ e^- \rightarrow J/\Psi \eta_c] = (17.6 \pm 2.8 \pm 2.1) fb \) and the Belle’s data,
\[ \sigma[e^+ e^- \rightarrow J/\Psi \eta_c] = (25.6 \pm 2.8 \pm 3.4) fb \] [1].

REFERENCES: