Mass spectrum of singlet and triplet $c\bar{b}$ meson states in non-relativistic quark model

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Introduction

Quark physics is a rich and challenging field for both theoretical and experimental research. Experimentally huge accelerators generating high energy particles are needed to study any interaction that can probe the physics at quark level. On the theoretical side also, a model for quarks should ultimately explain the characteristic features of the entire hadron family. The latter include predictions of the rest masses of the baryons and mesons, their decay and reaction mechanisms and their bound state behavior.

The complexity of quantum chromodynamics in the non-perturbative domain in spite of the progress of lattice QCD forces one to resort to models in order to be able to describe the phenomena of intermediate energy physics. Constituent quark models are a crucial element for the understanding of non-perturbative QCD and of the basic mechanisms which underlay the spectroscopy of hadrons. While they have already been proven successful in the past in the description of both hadronic masses and decay widths, they need to be challenged in face of new experimental data such as BES, E835, CLEO, BaBar, Belle, CDF, DO, NA60 etc. This offers a great challenge to the phenomenology to device a theoretical framework under one unified model.

The non-relativistic quark models (NRQM) are a class of phenomenological models developed to explain the hadron interactions. [1-2]. In studying mesons in the frame work of NRQM, one usually assumes that the $q\bar{q}$ wave function is a solution of a non relativistic Schrödinger equation.

The Hamiltonian of these quark models usually contains three main ingredients: the kinetic energy, the confinement potential and a hyperfine interaction term which has often been taken as an effective one-gluon-exchange potential (OGEP)[3]. The OGEP in our model is obtained from non relativistic reduction of the Fermi-Breit interaction.

The Hamiltonian employed in our model is given by [4],

$$H = K + V_{\text{CONF}}(\vec{r}) + V_{\text{OGEP}}(\vec{r})$$

where

$$K = \sum_{i=1}^{2} \left( M_i + \frac{P_i^2}{2M_i} \right) - K_{\text{CM}}$$

where $M_i$ and $P_i$ are the mass and momentum of the $i^{th}$ quark. $K$ is the sum of the kinetic energies including the rest mass minus the kinetic energy of the CM of the total system.

The potential energy part consists of confinement term $V_{\text{CONF}}$, the residual interaction $V_{\text{OGEP}}$.

The confinement term represents the non-perturbative effect of QCD that confines quarks within the colour singlet system and is taken to be linear

$$V_{\text{CONF}}(\vec{r}) = -a_{\lambda} \vec{r} \cdot (\vec{\lambda}_i \times \vec{\lambda}_j)$$

where $a_{\lambda}$ is the confinement strength. The term $\vec{r}$ here and elsewhere in the thesis stands for the relative distance between the quarks. The confinement potential $V_{\text{CONF}}$ is entirely central.
in nature. Here, $\overleftrightarrow{\lambda}_i$ and $\overleftrightarrow{\lambda}_j$ are the generators of the colour SU(3) group for the $i^{th}$ and $j^{th}$ quark.

The following central part of two-body potential due to OGEP is usually employed [3],

$$V_{\text{OGEP}}(r) = \frac{\alpha}{4} \overleftrightarrow{\lambda}_i \cdot \overleftrightarrow{\lambda}_j \left( \frac{1}{r} \frac{\pi}{MM_\mu} \left( 1 + \frac{2}{3} \vec{\sigma}_i \cdot \vec{\sigma}_j \right) \right) \delta(r)$$

where the first term represents the residual Coulomb energy and the second term the chromo-magnetic interaction leading to the hyperfine splitting. $\vec{\sigma}_i$ is the Pauli spin operator and $\alpha_s$ the quark-gluon coupling parameter.

The quark-quark interaction takes into account QCD perturbative effects by means of the one-gluon- exchange potential and the most important non perturbative effects through the confinement potential.

## Results and discussions

The $B_c$ pseudoscalar meson was originally seen in semileptonic decay channels at FNAL. In May 2005 the CDF collaboration announced an accurate determination of the $B_c$ mass. The state was seen in $14.6\pm4.6$ events in the $J/\psi\pi^\pm$ mode with the mass of $6285.7\pm5.3\pm1.2$ MeV. Experimental confirmation of $B_c^*$ meson has not been done yet. Using the non relativistic quark model we have calculated the masses of singlet and triplet $c\bar{b}$ meson states respectively. In our calculation we have expressed the product of quark-antiquark oscillator wave functions in terms of oscillator wave functions corresponding to the relative and centre-of-mass coordinates (CM). The calculated values of masses for both singlet and triplet mesons are given in the table 2. Table 1 gives the parameters used in our model. A comparison is made with the calculated masses of $B_c^*$ in case of the other models in table 3.

## Conclusions

The phenomenological non-relativistic quark model (NRQM) has been employed to obtain the masses of $B_c$ and $B_c^*$ states. The Hamiltonian used in the investigation has kinetic energy, confinement potential and one-gluon-exchange potential (OGEP). The total energy or the mass of the meson is obtained by calculating the energy eigen values of the Hamiltonian in the harmonic oscillator basis. An overall agreement is obtained with the experimental masses [5].

## References


### Table 1 Parameters in the model

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<th>Parameter</th>
<th>Value</th>
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<tr>
<td>$b$</td>
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<tr>
<td>$M_c$</td>
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<td>$M_b$</td>
<td>4300 MeV</td>
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<td>$\alpha_s$</td>
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### Table 2 Masses of $c\bar{b}$ mesons (MeV)

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<th>Meson</th>
<th>PDG</th>
<th>Our Model</th>
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<tbody>
<tr>
<td>$^2S_0$</td>
<td>$B_c$</td>
<td>$6276\pm4$ [5]</td>
<td>6278.25</td>
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<tr>
<td>$^2S_1$</td>
<td>$B_c^*$</td>
<td>6284-6357*</td>
<td>6291.03</td>
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*range of predictions from various papers

### Table 3 Comparison of $B_c^*$ mass with other model calculations

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<tr>
<td>$B_c^*$</td>
<td>6340±20</td>
<td>6318</td>
<td>6310</td>
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