Effect of nuclear reaction rates on primordial abundances

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Introduction
The theoretical predictions of the primordial abundances of elements in the big-bang nucleosynthesis (BBN) are dominated by uncertainties in the input nuclear reaction rates. We investigate the effect of modifying these reaction rates on light element abundance yields in BBN by replacing the thirty-five reaction rates out of the existing eighty-eight. We have studied these yields as functions of evolution time or temperature. We find that using these new reaction rates results in only a little increase in helium mass fraction over that obtained previously in BBN calculations. This allows insights into the role of the nuclear reaction rates in the setting of the neutron-to-proton ratio during the BBN epoch. We observe that most of these nuclear reactions have minimal effect on the standard BBN abundance yields of $^6$Li and $^7$Li.

Thermonuclear reaction rates
The twelve most important nuclear reactions which affect the predictions of the abundances of the light elements [e.g. $^4$He, $^3$He, $^7$Li] are $n$-decay, $p(n,\gamma)d$, $d(p,\gamma)^3$He, $d(d,n)^3$He, $d(d,p)t$, $^3$He$(n,p)t$, $t(d,n)^4$He, $^3$He$(d,p)^4$He, $t(\alpha,\gamma)^7$Be, $^7$Be$(n,p)^7$Li and $^7$Li$(p,\alpha)^4$He. Instead of cross sections $\sigma$, the nuclear reaction inputs to BBN take the form of thermal rates. These rates are computed by averaging nuclear reaction cross sections over a Maxwell-Boltzmann distribution of energies. Maxwellian-averaged thermonuclear reaction rate $<\sigma v>$ at some temperature $T$, is given by the integral

\[
<\sigma v> = \left[ \frac{8}{\pi \mu (k_B T)^3} \right]^{1/2} \int \sigma(E) \exp(-E/k_B T) dE,
\]

where $E$ is the centre-of-mass energy, $v$ is the relative velocity, $\mu$ is the reduced mass of the reactants and $k_B$ is Boltzmann constant. At low energies (far below Coulomb barrier) where the classical turning point is much larger than the nuclear radius, the barrier penetrability can be approximated by $\exp(-2\pi \zeta)$ so that the charge induced cross section can be decomposed into

\[
\sigma(E) = \frac{S(E) \exp(-2\pi \zeta)}{E}
\]

where $S(E)$ is the astrophysical $S$-factor and $\zeta$ is the Sommerfeld parameter, defined by

\[
\zeta = \frac{Z_1 Z_2 e^2}{\hbar v}
\]

where $Z_1$ and $Z_2$ are the charges of the reacting nuclei in units of elementary charge $e$. Except for narrow resonances, the $S$-factor $S(E)$ is a smooth function of energy, which is convenient for extrapolating measured cross sections down to astrophysical energies. The neutron induced reaction cross sections at low energies can be written as

\[
\sigma(E) = \frac{R(E)}{v}
\]

facilitating extrapolation of the measured cross sections down to astrophysical energies, where $R(E)$ is a slowly varying function of energy and is similar to an $S$-factor. In the case of a narrow resonance, the resonant cross section $\sigma_r(E)$ is generally approximated by a Breit-Wigner expression.

Calculations and Results
A comprehensive study of the effect of modifying thirty-five nuclear reaction rates (listed

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TABLE I: Yields at test ($\eta_{10} = 3.162$) and WMAP ($\eta_{10} = 6.19 \pm 0.15$) [4] baryonic densities.

<table>
<thead>
<tr>
<th>Element</th>
<th>Test</th>
<th>Wagoner</th>
<th>Wagoner</th>
<th>Observations</th>
<th>Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^4\text{He}$</td>
<td>0.2410</td>
<td>0.2411</td>
<td>0.2479</td>
<td>0.2479</td>
<td>0.232-0.258[5] × 10$^{-5}$</td>
</tr>
<tr>
<td>$D/\text{H}$</td>
<td>7.250</td>
<td>7.277</td>
<td>2.519</td>
<td>2.563</td>
<td>2.82$^{+0.26}_{-0.19}$[6] × 10$^{-5}$</td>
</tr>
<tr>
<td>$^3\text{He}/\text{H}$</td>
<td>1.546</td>
<td>1.613</td>
<td>1.033</td>
<td>1.058</td>
<td>0.9-1.37[7] × 10$^{-5}$</td>
</tr>
<tr>
<td>$^7\text{Li}/\text{H}$</td>
<td>1.268</td>
<td>1.367</td>
<td>4.627</td>
<td>5.019</td>
<td>1.1±0.1[8] × 10$^{-10}$</td>
</tr>
</tbody>
</table>

FIG. 1: Plots of all the abundances with modified Maxwellian-averaged thermonuclear reaction rates. The curve marked ‘rest’ represents sum of all the abundances of nuclei higher than $^7\text{Be}$.

in Table-I of Ref. [1] on primordial nucleosynthesis is performed. The impact on BBN of the recent compilation of thermonuclear reactions rates [2, 3] does not make large overall changes. These modified rates do affect the magnitude of these predictions at intermediate times for cases such as $^4\text{He}$ or $^7\text{Li}$ but the final values of these predictions remain almost same. The calculations are performed with a test value for the baryons to photons ratio $\eta = \eta_{10} \times 10^{-10} = 3.162 \times 10^{-10}$ which reproduces the observed $^7\text{Li}$ abundance for the standard BBN. In Fig.1 plots of abundances of all the elements with respect to the number of $H$ nuclei are shown as functions of evolution time. In Table-I, the comparison between BBN abundances deduced using the test value ($\eta_{10} = 3.162$) and from the WMAP [4] results ($\eta_{10} = 6.19$) is provided.

Summary and Conclusion

In summary, we find little effect on the standard BBN abundance yields by replacing the Maxwellian-averaged thermonuclear reaction rates by new ones. The chances of solving either of the lithium problems by conventional nuclear physics means are unlikely and, if these problems stand up to future observations, we may be forced into just such non-standard BBN scenarios.

References