Level Density of $^{16}$O

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Introduction

Nuclear level densities are of special importance in Nuclear Physics. The number of excited levels of a nucleus is known to increase very rapidly with increasing excitation energy. This fact reflects the increasing number of degrees of freedom that can be excited. If the excitation energy is not too small these degrees of freedom are mainly due to the single particle levels. Level densities represent a very important ingredient in statistical model calculations of nuclear reaction cross section, and are needed in many applications from astrophysical calculations to fission and fusion reactor designs. Most of the existing experimental data are based on measuring level densities at an energy close to the neutron binding energy, by counting the number of neutron resonances observed in low-energy neutron capture.

The level densities can be determined at lower excitation energies by directly counting the observed excited states. There are a number of theoretical approaches at a microscopic level, considering shell effects, pairing correlations and collective effects, but their use in practical applications is rather complicated[1]. At present the level density for practical applications is calculated mainly on the basis of the Fermi gas[2] and Gilbert-Cameron[3] formulae with adjustable parameters which are found from experimental data on neutron resonance spacing and the density of low lying discrete levels. In GDR studies, particularly damping width have been studied in several nuclei for excitation energy ranging from 50 to 200 MeV, and it has been shown that the analysis of experimental data is very sensitive to the dependence of level density on excitation energy[4].

From the last decade, Oxygen and its isotopes get much attention in the research area such as determination of $\alpha$-decay widths[5], shell gap at $N$=16[6], excited states and shell structure[7-9]. Generally, light nuclei are having more astrophysical importance; we have studied a light nucleus $^{16}$O in the context of level density and its possible structural existence at zero spin.

Methodology

The present work is to extend the Monte Carlo method of actual counting [10] of complexions or configurations that yield the same energy $E$ and spin $J$ of the whole nucleus containing $N_o$ available states for the $n$ particles in the system. For light nuclei this method is more realistic provided shell model and single particle levels incorporating proper parameters are used. The results presented here indeed reflect the shift in the most probable energy and spin in a system when the entropy $S$, energy $E$ and spin $J$, surface is drawn. It should be noted that these spins are purely due to single particle excitation, not collective excitation. The method starts with the generation of single particle states $\varepsilon_i$ as a function of $z$ component of the single particle spins $m_i$ up to $N=11$ shells using the Nilsson model with Lund parameters, $\kappa$ and $\mu$ for the sake of simplicity. The single particle eigenvalues $\varepsilon_i$ are different for protons and neutrons ($\varepsilon_i^Z$ and $\varepsilon_i^N$). The diagonalisation of the Hamiltonian is done using cylindrical basis states with Hill-Wheeler deformation parameters ($\theta, \delta$). The expression for the level density $\rho(E^*)$ is

$$\rho \left( E^* \right) = \left( kT N_o \ln 2 \right)^{-1} \exp \left[ S \left( E^* \right) \right],$$

where $N_o$ is the total number of eigen states used in the model space and the entropy,

$$S(E) = \sum \left[ \left( 1 - n_i \right) \ln \left( 1 - n_i \right) + n_i \ln n_i \right],$$

where $n_i$ are the single particle occupation probabilities. To study collective rotations the method is repeated with cranked Nilsson model.
levels with the rotational frequency $\Omega = 0.0\hbar\omega$, 0.05$\hbar\omega$ and 0.1$\hbar\omega$, where $\omega$ is the oscillator frequency.

The particles, neutrons/protons are allowed to fill up the states in a random fashion. Suppose in the $K$th configuration if $n_{iK}$ is the single particle occupation probability then the total particle number $n$ is, $n = \sum n_{iK}$, the corresponding total energy for the $K$th configuration $E = E_K = \sum n_{iK} \varepsilon_i$. Here $K$ is the index for configurations and ‘i’ represent single particle levels. The different configurations generated in the $N_0$ particle states ($N_0 = 20$) by $n$ number of particles ($N_0 \geq n$) for a given $J$ and $E$ are then counted to get the total number of configurations $W(E,J)$ for each $J$ and $E$ [11]. The entropy $S(E,J)$ are then obtained using the equation, $S(E,J) = k \ln W(E,J)$, where $k$ is the Boltzman constant.

The results obtained for $^{16}$O is presented here and our analysis is based on the maximum density of states and its shift as the angular momentum decreases or increases. The most probable macroscopic state will correspond to the center marked as $W_{\text{max}}$ in contour graph and may reflect $S_{\text{max}}$ in experimental observations and if $\exp(2(aE^*)^{1/2})$ is for extracting ‘a’,

$$2(aE^*)^{1/2} = S_{\text{max}} = \log(W_{\text{max}}); \quad a = S^2/4E^*.$$

**Results and Discussion**

A prolate deformed shape at the spin $0\hbar$ is observed for the nucleus $^{16}$O at deformation $\delta=0.5$ since $W_{\text{max}}$ is obtained at this deformation. In CNM (Cranked Nilsson Model) when we fix the rotational frequency $\Omega = 0.05\hbar\omega$ the $W_{\text{max}}$ is obtained at $\delta=0.6$, and when $\Omega = 0.1\hbar\omega$, $W_{\text{max}}$ is at $\delta=0.6$. Since the ground state prolate deformation obtained is equivalent to superdeformed state one can expect an (2$\alpha$-2$\alpha$) cluster configuration or ($^8\text{Be}-^8\text{Be}$) cluster type, but detailed investigations are necessary for the existence of a region of angular momentum where the linear chain configuration is stabilized[12]. The entropy, $S(E)/K$, at this ground state deformation where maximum configuration obtained is 5.46879, 6.28059 and 6.01253 for the given excitation energy 36.0153MeV, 45.3571MeV and 49.1061MeV respectively.

![Fig.1. Contour plot of $W_{\text{max}}$ for $^{16}$O](image)

The level density or entropy is maximum at oblate deformation (0 =-180° and $\delta=0.4$) at spin $J=5\hbar$ when $\Omega = 0.0\hbar\omega$, at $W_{\text{max}} = 0.518035\times10^6$; shown in fig.1. When $\Omega = 0.05\hbar\omega$, the $W_{\text{max}} = 3.96229\times10^6$ at oblate deformation (0 =-180° and $\delta=0.4$). Similar effect is observed while increasing the rotational frequency $\Omega = 0.1\hbar\omega$, that $\delta=0.2$ and $W_{\text{max}} = 4.89342\times10^6$. So the system $^{16}$O would have a greater probability of being oblate at $\Omega = 0.0\hbar\omega$, 0.05$\hbar\omega$, and 0.1$\hbar\omega$ for the given excitation energy 38.9653MeV, 45.3571MeV and 49.1061MeV respectively.

**References**