Temperature-dependence in Seeger’s liquid drop energy
and the dynamical cluster-decay model

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Introduction

Seeger’s semi-empirical mass formula \cite{1} is revisited for two of its constants (bulk constant \(\alpha(0)\) and neutron-proton asymmetry constant \(a_a\)) readjusted to obtain the ground-state (g.s.) binding energies of nuclei within a precision of <1.5 MeV and for nuclei up to \(Z=118\). The aim is not to obtain a new parameter set of Seeger’s liquid drop energy \(V_{LDM}\), but to include the temperature T-dependence on experimental binding energies \cite{2}. The T-dependence, in the constants of \(V_{LDM}\), is introduced as per the work of Davidson \textit{et al.} \cite{3}, where the pairing energy \(\delta(T)\) is modified as per new calculations on compound nucleus (CN) decays. The newly fitted constants of \(V_{LDM}\) at \(T=0\) are made available in a (small) tabular form for use of other workers interested in developing computer codes on nuclear dynamics of hot and rotating nuclei. The main purpose of this work is to give a procedure for calculating the fragmentation potentials of nuclei at the incident energies used in heavy ion reactions, i.e., using the T-dependent experimental binding energies, as demonstrated here for the decay of CN \(^{56}\text{Ni}\)\textsuperscript{*}.

Methodology

The collective fragmentation potential \(V(\eta, R, T)\) that brings in the structure effects of the CN in to the dynamical cluster-decay model (DCM) of Gupta and collaborators \cite{4, 5}, is calculated according to the Strutinsky renormalization procedure \((B = V_{LDM} + \delta U)\), using the T-dependent liquid drop model energy \(V_{LDM}(T)\) of Davidson \textit{et al.} \cite{3} and the empirical shell corrections \(\delta U\) of Myers and Swiatecki \cite{6}, for spherical nuclei, also made T-dependent to vanish exponentially with \(T_0=1.5\) MeV. It is given as

\[
V(\eta, R, T) = \sum_{i=1}^{2}[V_{LDM}(A_i, Z_i, T)] + \\
\sum_{i=1}^{2} [\delta U_i] \exp(-T^2/T_0^2) + V_C(Z_i, \beta_{\lambda_i} , \theta_i, T) + \\
V_P(A_i, \beta_{\lambda_i}, \theta_i, T) + V_l(A_i, \beta_{\lambda_i}, \theta_i, T),
\]

where nuclear proximity \(V_P\), Coulomb \(V_C\) and the angular momentum \((l)\) dependent \(V_l\) potential are for oriented nuclei and are also T-dependent. The \(V_{LDM}\) here, based on the semi-empirical mass formula of Seeger \cite{1}, is

\[
V_{LDM}(A, Z, T) = \alpha(T)A + \beta(T)A^{\frac{2}{3}} + \\
\left(\frac{\gamma(T) - \eta(T)}{A^{\frac{2}{3}}} \right) \left( I^2 + 2|I| \right) + \left( \frac{Z^2}{r_0(T)A^{\frac{2}{3}}} \right) \\
\times \left( 1 - \frac{0.7636}{Z^{\frac{4}{3}}} - \frac{2.29}{|r_0(T)A^{\frac{2}{3}}|} \right) + \\
\delta(T)f(Z, A)/A^{\frac{2}{3}},
\]

with \(I = a_a(Z - N), a_a = 1\) and, respectively, for even-even, even-odd and odd-odd nuclei, \(f(Z, A) = (-1, 0, 1)\).

Seeger \cite{1} fitted the constants of \(V_{LDM}\) to the ground state \((T=0)\) binding energies (BES) of some 488 nuclei available at that time (in 1961). These constants certainly require modification due to the availability of large amount of data \cite{2, 7} on ground-state BES. The temperature dependence of the constants of \(V_{LDM}\) in Eq. (2) are given in Fig. 1 of \cite{3}.

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Calculations and Discussions

In Table 3.1 of Ref. [5], we have presented the fitted constants for the experimental [2] and theoretical [7] BEs. The fitted constant \(\alpha(0)\) is working as an overall scaling factor, and \(a_n\) controls the curvature of the experimental parabola, depicted in Fig. 1 for \(Z=97\) nuclides. Fig. 1 shows the excellent agreement between the present fits (crosses and down open triangle) corresponding to experimental data [2] (solid circles) and theoretical data [7] (open circle), respectively. The fits are obtained between 0-1.5 MeV of the available experimental and theoretical data. The calculated BEs using the 1961 Seeger’s constants are also shown in Fig. 1 (hollow square), which shows the requirement and extent of fitting required.

Next, we consider an application of the re-adjusted \(V_{LDM}\) with an idea to impress upon the need and to propose here at least a partially modified variation of the pairing constant \(\delta\) with temperature \(T\), as compared to that of Davidson et al. [3]. Fig. 2 shows the fragmentation potential \(V(A)\) for the decay of \(^{56}\)Ni\(^*\) (a complete mass spectrum) into light particles (LPs) and intermediate mass fragments (IMFs) at \(T=3.60\) MeV for two different \(\ell\) values (\(\ell=0\) and 36 \(h\)), compared with one at \(T=0\) MeV for \(\ell=0\) \(h\). We notice that at \(T=0\) MeV for \(\ell=0\) \(h\), the pairing effects are very strong since all the even-even fragments lie at potential energy minima. On the other hand, if we include the temperature effects as per prescription of Davidson et al., we find that \(\delta=0\) MeV in \(V_{LDM}\) for \(T>2\) MeV, and hence in Fig. 2 for \(T=3.60\) MeV, \(\delta=0\) MeV, the odd-odd fragments, like \(^{10}\)B, \(^{14}\)N, \(^{18}\)F, etc., become equally probable as the even-even fragments, since minima are now equally stronger. However, if we empirically choose \(\delta=9.5\) MeV for \(T=3.60\) MeV (for the best fit to IMFs data), the situation becomes again favorable. In other words, Fig. 2 for \(T=3.60\) MeV, \(\delta=9.5\) MeV shows once again that the even-even fragments, like \(^{12}\)C, \(^{16}\)O, etc., are favored over odd-odd \(^{14}\)N, \(^{18}\)F, etc. These calculations lead us to modify the variation of \(\delta\) as function of \(T\). Apparently, many more calculations are needed.

References