Study of shape phase transition at N=60 in Zr and Pd

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The level structure of Zr, Mo, Ru and Pd in the region of A=100 has been of much interest recently [1]. At N=60 the level structure varies rather sharply. Below N=60 the structure is vibrator like and at N>60, the structure corresponds to a rotor. The energy of $^2_1^+$ drops sharply at N=60 [1]. But this drop in energy is very much dependent on the Z value as well. The sharpest drop is for Sr and it decreases for Zr, Mo, Ru, and Pd successively. This represents the strong effect of the g$_{7/2}$ sub-shell closure at N=58. Here the collectivity builds up quickly with the addition of valence neutrons. The effect of the valence protons is exhibited in the Z-dependent behavior.

The ground band itself carries much information on the collectivity. Thus it is of interest to reproduce these energy variations in a suitable energy formula. The single term power index formula [2] has the advantage that only two terms of the spectrum can yield the values of the two parameters of the formula, which can be used to predict the energies at higher spins.

$E_I = a I^b$.

Thus we have studied the level structure of $^{98-110}$Pd and $^{100-104}$Zr, using this formula. This encompasses the N=60 point as well.

This equation can be solved easily by taking the ratio for I=4 and 2. This eliminates ‘a’, then taking log, one gets the index ‘b’. This is repeated for all available spin I in the nucleus. Then one finds an average of all ‘b’. This average ‘b’ is used to determine the values of ‘a’ for all spin I. Using the average ‘b’ and average ‘a’, one recalculate the level energies. A root mean square deviation of the energies from the experiment determines the goodness of fit of energies to the two empirical parameters ‘a’ and ‘b’. However, it depends on the number of levels as well.

Results

The values of ‘b’ versus spin I are plotted for each of the Pd and Zr isotopes in Fig. 1 and 2 respectively. In fig. 1 we study the variation of ‘b’ with N for Pd isotopes. Here, the N=62-64 isotopes have large ‘b’, steady with spin I. At N=58, 60 fewer data were available, and not much different from those at N=62-64. At N=56 there is some drop in ‘b’ which drops sharply at N=54. At N=52, an almost spherical nucleus, not only ‘b’ is small, it varies with spin. Average value is almost 1.0 corresponding to a spherical vibrator. The values of average ‘b’ and average ‘a’ are shown in the Table, along with RMSD values for energies.

As seen in Fig.2 for Zr, ‘b’ is highest at N=64 and is near 1.8, maximum being ≤2 for good rotors. Also it is nearly constant with spin for N=62, 64. At N=60 in $^{100}$Zr, there is some variation of ‘b’ with spin. The increasing ‘b’ reflects the increase in deformation with spin. That is it is a beta soft nucleus. This is expected at the shape transition point. At N=58, there is a sharp rise in $E(2_1^+)$ and drop in the value of ‘b’. In fact it is the lowest value for Zr. But not sufficient levels in gs band are available. So no calculation could be done for $^{94-98}$Zr.
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Fig. 1. Plot of index ‘b’ vs spin I for Pd.

Fig. 2. Plot of index ‘b’ vs. spin I for Zr.

<table>
<thead>
<tr>
<th>N</th>
<th>52</th>
<th>54</th>
<th>56</th>
<th>58</th>
<th>60</th>
<th>62</th>
<th>64</th>
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<tbody>
<tr>
<td>Pd</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Av. b</td>
<td>0.861</td>
<td>1.090</td>
<td>1.217</td>
<td>1.263</td>
<td>1.269</td>
<td>1.273</td>
<td>1.327</td>
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<tr>
<td>Av. a</td>
<td>480.5</td>
<td>313.0</td>
<td>240.2</td>
<td>231.8</td>
<td>212.4</td>
<td>179.4</td>
<td>150.8</td>
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<tr>
<td>RMSD</td>
<td>178</td>
<td>32.8</td>
<td>48.2</td>
<td>12.3</td>
<td>7.3</td>
<td>34</td>
<td>38.61</td>
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<tr>
<td>Zr</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Av. b</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.481</td>
<td>1.686</td>
<td>1.732</td>
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<tr>
<td>Av. a</td>
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<td></td>
<td>77.2</td>
<td>47.4</td>
<td>42.2</td>
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<tr>
<td>RMSD</td>
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<td>93</td>
<td>24</td>
<td>41</td>
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</tr>
</tbody>
</table>

**Conclusion**

The use of single term expression for level energies in the study of shape phase transition at N=60 is illustrated here. The much sharper shape transition of Zr and slower for Pd is exhibited. There being only two free parameters one can study the nuclei with fewer levels as well. Further the constancy of ‘b’ with spin is also tested.

**References**
