Bubble Nuclei in Relativistic Mean Field Theory

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Introduction

Bubble nuclei are characterized by a depletion of their central density, i.e., the formation of the proton or neutron void and subsequently forming proton or neutron bubble nuclei. Possibility of the formation of bubble nuclei has been explored through different nuclear models and in different mass regions [1]. Advancements in experimental nuclear physics have led our experimental access to many new shapes and structures, which were inaccessible hitherto. In the present paper, we have studied the possibility of observing nuclear bubble in oxygen isotopes, particularly for $^{22}$O.

Mathematical Details

The axially deformed relativistic mean field theory has been used to study the bubble nuclei. Here we present one of the potential cases, which has been predicted to be bubble nuclei too. The RMF theory is well established and we refer [2] for details, however for sake of completeness, we sketch the mathematical details here in brief. The basic ingredient is the relativistic Lagrangian density for a nucleon–meson many-body system [2, 3]:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - \frac{1}{2}g_\rho \bar{\psi}\rho\psi - \frac{1}{2}g_\omega \bar{\psi}\omega\psi - \frac{1}{2}g_{\phi} \bar{\psi}\phi\psi - \frac{1}{4}f_{\sigma}^2 (\sigma^\mu\nu\sigma^\rho\sigma_\rho)^\mu\nu - \frac{1}{4}f_{\lambda}^2 (\lambda^\mu\nu\omega^\rho\sigma_\rho)^\mu\nu - \frac{1}{4}f_{\gamma}^2 (\gamma^\mu\nu\phi^\rho\sigma_\rho)^\mu\nu - \frac{1}{4}f_{\lambda}^2 (\lambda^\mu\nu\sigma^\rho\omega_\rho)^\mu\nu - \frac{1}{4}f_{\gamma}^2 (\gamma^\mu\nu\phi^\rho\omega_\rho)^\mu\nu$$

The symbols denote the standard terms, as described in another paper in the same conference. Though, there exist a number of parameter sets for solving the standard RMF Lagrangians, we have used the NL–SH parameter set in this work, which has been successfully used for many other studies in a different mass regions, for RMF calculations.

Results and Discussion

We see from the contour plots, for the total nucleon distribution, as displayed in figure 1 (here, the full black portion is the maximum-density region and the full white one is the zero-density region), that the central part of the nucleus is less dense than the peripheral region.

Figure 1. Contour plot for the neutron, proton and total density distribution for $^{16,22,24}$O. The first column is for neutron density, second for proton density and third for total density distribution. Similarly, first, second, third and fourth rows represent $^{16}$O, $^{22}$O and $^{24}$O, respectively. Here, the full black portion is the maximum-density region and the full white one is the minimum-density region. The contours are drawn in a square box of size $\approx 5.5$ fm.

This type of density distribution can be interpreted as a low-density nucleon gas...
surrounded by a high-density, thick ring of nucleons, giving rise to a ‘bubble’-like structure
(a nucleus which is hollow at the centre). We note from the individual density distributions of
neutrons and protons that there is depletion of density in the central region for the neutron
densities in the case of $^{16-22}$O but not for $^{14,24}$O, and for the proton densities in the case of $^{16-24}$O
but not for $^{14}$O. Hence, to have a quantitative measure of the bubble effect for all the nuclei
under consideration, we have calculated the depletion fraction for the neutron, proton as well
as for the total nucleon distribution as given in table 1. The depletion fraction (in percentage)
can be defined as $^{[4]}$

$$\text{(D.F.)}_\alpha = 100 \times \frac{(\rho_{\text{max}})_\alpha - (\rho_{\text{cen}})_\alpha}{(\rho_{\text{max}})_\alpha}$$

where $\rho_{\text{max}}$ and $\rho_{\text{cen}}$ represent the values of the maximum and central densities, respectively, and $\alpha$ denotes the neutron, proton or the total nucleonic matter.

Table 1. The depletion fraction as a measure of depletion of central density for the neutron,
proton and total nucleon distribution for $^{14-24}$O.

<table>
<thead>
<tr>
<th>Nuclei</th>
<th>(D.F.) neutron</th>
<th>(D.F.) proton</th>
<th>(D.F.) total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{14}$O</td>
<td>0.4444</td>
<td>1.5940</td>
<td>1.032405</td>
</tr>
<tr>
<td>$^{16}$O</td>
<td>12.3839</td>
<td>12.2503</td>
<td>10.9677</td>
</tr>
<tr>
<td>$^{18}$O</td>
<td>16.5148</td>
<td>16.2762</td>
<td>16.4002</td>
</tr>
<tr>
<td>$^{20}$O</td>
<td>21.7054</td>
<td>21.0593</td>
<td>21.4033</td>
</tr>
<tr>
<td>$^{22}$O</td>
<td>24.5425</td>
<td>25.3197</td>
<td>24.8977</td>
</tr>
<tr>
<td>$^{24}$O</td>
<td>2.6123</td>
<td>19.3421</td>
<td>10.0175</td>
</tr>
</tbody>
</table>

It is apparent from the table that although there is density depletion in the central region for
other isotopes too, $^{22}$O can be considered as the best candidate to see the bubble effect among the
nuclei. The possibility of $^{22}$O being neutron bubble nuclei has been examined earlier too $^{[1]}$, where a significant model dependence was observed, thereby calling into question the bubble structure of $^{22}$O. Also, we see that there is a sharp increase in the total reaction cross section for $^{22-24}$O, suggesting a distinct change in the nuclear structure in the vicinity of $^{22}$O.

It is of particular interest to inquire how the density distributions obtained here may be
affected by pairing interactions. In order to take care of the pairing effects in this work, we use
the constant gap for the proton and neutron, as given in $^{[3]}$: $\Delta_p = RB_s e^{-sI^2/A^{1/3}}$ and $\Delta_n = R B_s e^{-tI^2/A^{1/3}}$, with $R = 5.72$, $s = 0.118$, $t = 8.12$, $B_s = 1$ and $I = (N-Z)/(N+Z)$. This type of prescription for pairing effects, both in the RMF and Skyrme-based approaches, has already been used by us and many other authors $^{[5]}$.

**Conclusion**

In summary, we have looked into the possibility of observing the bubble effect in $^{22}$O in conjunction with RMF theory. However, the possibility of $^{22}$O being bubble nuclei needs further investigation experimentally as well as theoretically.

It is well established that RMF-based approaches determine the ground-state observables and nuclear density distributions of nuclei quite successfully and could further be used for several other potential candidates to know more about such peculiar nuclear structure. Also, it has been shown that in a bubble configuration, the spin-orbit splitting of low lying doublets is sometimes reversed, and that this effect is especially pronounced for levels with low angular momentum. The occurrence of a bubble is also explained by an inversion of $s_{1/2}$ state with the state usually located above. These effects deserve further detailed study with other potential candidates, such as $^{36}$Ar, $^{36}$S, $^{34}$Si, $^{138}$Ce and $^{200}$Hg $^{[6]}$.

**References**