Lattice based equation of state and transverse momentum spectra of identified particles in ideal and viscous hydrodynamics

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Relativistic hydrodynamics provides a convenient tool to analyse relativistic heavy ion collision data. It is assumed that a fireball is created in the collisions. Assuming a local thermal equilibrium achieved after a certain time \( \tau_i \) hydrodynamics can be applied. If the macroscopic properties of the fluid are known at the equilibration time \( \tau_i \), the relativistic hydrodynamic equations can be solved to give the space-time evolution of the fireball till a given freeze-out condition such that interaction between the constituents is too weak to continue the evolution. Using suitable algorithm (e.g. Cooper-Frye) information at the freeze-out can be converted into particle spectra and can be directly compared with experimental data. Thus, hydrodynamics, in an indirect way, can characterize the initial condition of the medium produced in heavy ion collisions. Hydrodynamics equations are closed only with an equation of state (EOS). It is one of the most important inputs of a hydrodynamic model. Through this input macroscopic hydrodynamic models make contact with the microscopic world and one can investigate the possibility of phase transition in the medium. Most of the hydrodynamical calculations are performed with EOS with a 1st order phase transition. However, lattice simulations for \( \mu_B = 0 \) indicate that the confinement-deconfinement transition is neither a 1st nor a 2nd order phase transition, rather a cross-over at \( T_{co}=196 \) MeV. It is then important that lattice based EOS are used in hydrodynamic analysis of RHIC data, when we are trying to verify the lattice prediction about confinement-deconfinement transition.

In hydrodynamics, initial energy density or temperature of the fluid is obtained by fitting experimental data on particle production, e.g. pion multiplicity, \( p_T \) spectra etc., which essentially measure the final state entropy. Unlike in ideal fluid evolution, where initial and final state entropy remains the same, in viscous fluid evolution entropy is generated. Consequently, to produce a fixed final state entropy, viscous fluid require less initial entropy density (or energy density) than an ideal fluid. Assuming that in Au+Au collisions, a baryon free fluid is produced, transverse momentum spectra of identified particles (\( \pi, K, p \) and \( \phi \)), in evolution of ideal and viscous fluid is studied. Hydrodynamic evolution is governed by a lattice based equation of state (EOS), with a cross-over at \( T_{co}=196 \) MeV. Ideal or viscous fluid was initialised to reproduce \( \phi \) meson multiplicity in 0-5% Au+Au collisions. The energy momentum conservation and relaxation equation for shear stress in Israel-Stewart second order formalism is

\[
\partial_\nu T^{\mu\nu} = 0, \quad (1)
\]

\[
D\pi^{\mu\nu} = -\frac{1}{\tau_\pi}(\pi^{\mu\nu} - 2\eta\nabla^\mu u^\nu) - [u^\mu\pi^{\nu\lambda} + u^\nu\pi^{\mu\lambda}]Du^\lambda. \quad (2)
\]

Eq.1 is the conservation equation for the energy-momentum tensor, \( T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu} + \pi^{\mu\nu}, \varepsilon, p \) and \( u \) being the energy density, pressure and fluid velocity respectively. \( \pi^{\mu\nu} \) is the shear stress tensor. Eq.2 is the relaxation equation for the shear stress tensor \( \pi^{\mu\nu} \). In Eq.2, \( D = u^\mu \partial_\mu \) is the convective time derivative, \( \nabla^\mu u^\nu = \frac{1}{2}(\nabla^\nu u^\mu + \nabla^\mu u^\nu) - \frac{1}{2}(\partial_\mu u)(g^{\mu\nu} - u^\mu u^\nu) \) is a symmetric traceless tensor. \( \eta \) is the shear viscosity and \( \tau_\pi \) is the relaxation time. Assuming boost-invariance, Eqs.1 and 2 are solved in \( (\tau = \frac{1}{\tau_\pi} \text{MeV}) \)
FIG. 1: PHENIX data [2] on the centrality dependence of $\pi^+ + \pi^-$ in 0-60% Au+Au collisions. Experimental data beyond 0-5% are divided by a factor of 10 respectively. Four lines are hydrodynamic predictions from ideal($\eta/s=0.0$) and viscous ($\eta/s=0.08,0.12,0.16$) fluid respectively.

FIG. 2: Filled squares are the for the combined data sets as a function of viscosity to entropy ratio ($\eta/s$). The filled circles are the same when proton data are excluded. Experimental data only upto 30-40% collision centrality are included.

The goodness of fit can be best viewed in the $\chi^2$ analysis. In Fig.2, $\chi^2$ as a function of $\eta/s$ is shown. $p_T$ spectra of pions, kaons, phi and proton in 0-40% collisions are included in the $\chi^2$ analysis. As the Proton data is poorly described, we have also shown the $\chi^2$ values omitting the proton spectra. With or without the proton, best fit to data is obtained for viscosity to entropy ratio $\eta/s=0.08$. Ideal fluid also give comparable description. Description to the data is much poorer in viscous fluid evolution with $\eta/s \geq 0.12$. Identified spectra thus give a phenomenological upper limit of QGP viscosity, $\eta/s < 0.12$.

References