The $^3S_1$ and $^3D_1$ n-p scattering phase shifts using semiclassical wave equations

Arun K. Jain, Sudhir R. Jain, and B. N. Joshi
Nuclear Physics Division, Bhabha Atomic Research Centre, Mumbai - 400 085

A semiclassical treatment is developed for n-p scattering leading to decoupled equations (to order $O(h^2)$) even in the presence of tensor interaction. As the orbital angular momentum is not conserved with the tensor interaction the triplet states with the same angular momentum $J$ have two orbital angular momenta. For example, coupling the total spin $S = 1$ with two orbital angular momenta $L = 0$ and $L = 2$ produce a state with $J = 1$ and same positive parity. The second order differential equations coupling the $^3S_1$ state, $u(r)$ and $^3D_1$ state, $w(r)$ for the n-p scattering are

$$\frac{d^2u}{dr^2} + V_c(r)u - Eu = -\sqrt{8}V_t(r)w$$  

(1)

$$\frac{d^2w}{dr^2} - \frac{6u}{r^2} + [V_c(r) - 2V_t(r)]w - Ew = -\sqrt{8}V_t(r)u$$  

(2)

where $M$ is the reduced mass of the n-p scattering state, $V_c(r)$ and $V_t(r)$ are the central and tensor parts of the n-p interaction. For the Reid soft core potential, we can write

$$V(r) = V_c + V_t S_{12} + V_{LS}(L,S)$$  

(3)

with

$$V_c(r) = -10.463e_1(x) + 105.468e_2(x)$$

(4)

$$V_t(r) = -10.463$$

$$\left[1 + \frac{3}{x} + \frac{3}{x^2}\right]e_1(x) - \left[\frac{12}{x^2} + \frac{3}{x^2}\right]e_4(x)$$

$$+ 351.77e_4(x) - 1673.5e_6(x)$$

$$V_{LS} = 708.91e_4(x) - 2713.1e_6(x)$$

where $e_n(x) = e^{-nx}/x$ and $x = 0.7r$. Writing Eqs.(1) and (2) in a matrix eigenvalue form,

$$\mathbf{H}\Psi = E\Psi$$  

(6)

with

$$\Psi = \begin{bmatrix} u \\ w \end{bmatrix},$$

$$\mathbf{H} = \left(-\frac{h^2}{M} \frac{d^2}{dr^2}\right) \mathbf{I} + \mathbf{V}(r),$$

$$\mathbf{V}(r) = \begin{bmatrix} V_c(r) & \sqrt{8}V_t(r) \\ \sqrt{8}V_t(r) & \frac{6b^2}{M} + V_c(r) - 2V_t(r) \end{bmatrix}$$

(7)

Here, $\mathbf{I}$ is an identity matrix. Potential matrix $\mathbf{V}$ being real-symmetric, the diagonalizing transformation is orthogonal $\mathbf{O}$, such that

$$\mathbf{O}(r)^T \mathbf{V}(r) \mathbf{O}(r) = \mathbf{v}(r) = \begin{bmatrix} v_+(r) & 0 \\ 0 & v_-(r) \end{bmatrix},$$

where $\mathbf{O}(r) = \begin{bmatrix} \cos \frac{\theta}{2} & \sin \frac{\theta}{2} \\ -\sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{bmatrix}$

with $\tan \theta = \frac{\sqrt{8}V_t(r)}{\frac{3b^2}{M} - V_t(r)}$  

(8)

$$v_\pm(r) = a(r) \pm b(r) \cos \theta(r) \mp \sqrt{8}V_t(r) \sin \theta(r)$$

where $a(r) = \frac{3b^2}{M} + V_c(r) - V_t(r)$ and $b(r) = -\frac{3b^2}{M} + V_t(r)$.

Since $-ib\frac{d}{dr}(= p_r)$ does not commute with $r$, there appears a vector potential $\mathbf{A}$ but we note that it is purely off-diagonal and carries an order of $\hbar$ with it. We take only terms of order $O(\hbar^0)$ and take the similarity-transformed Hamiltonian, $\mathbf{H}^{(1)} = \mathbf{O}^T \mathbf{H} \mathbf{O}$ to get diagonalized form of the Hamiltonian as $\mathbf{H}_{\mu \nu} = h^\mu \delta_{\mu \nu}$ where

$$\hbar^{(\pm)} =$$

$$\left(-\frac{h^2}{M} \right) \left[1 \pm \frac{h^2}{M(v_+(r) - v_-(r))} \left(\frac{d\theta(r)}{dr}\right)^2\right]$$

$$\frac{d^2}{dr^2} + v_\pm(r) + \frac{h^2}{8M} \left(\frac{d\theta(r)}{dr}\right)^2.$$  

(9)

Avilable online at www.sympnp.org/proceedings
With this,

\[ \psi^+(r) = u(r) \cos \frac{\theta(r)}{2} - w(r) \sin \frac{\theta(r)}{2}, \]
\[ \psi^-(r) = u(r) \sin \frac{\theta(r)}{2} + w(r) \cos \frac{\theta(r)}{2}. \]  

(10)

For large values of \( r \), we get \( \theta \sim 0 \). Therefore, as \( r \to \infty \),

\[ \psi^+(r) = u(r), \quad \psi^-(r) = w(r). \]  

(11)

These equations are numerically solved using the Runge-Kutta method and matching \( \psi(r) \) and their derivatives with the asymptotic scattering state wavefunctions the phase shifts are evaluated. Thus one obtains the phase shifts for \( ^3S_1 \) and \( ^3D_1 \) scattering states.

In Fig. ??, the results are in good agreement with large matrix diagonalizations results. Good agreement indicates the success of our semiclassical wave equations which are quite simple and physically appealing.

References