Trajectory calculations can tell us what happens to a light charged particle (LCP) after emission. Here we propose a method to calculate the trajectory of $^{10}$Be emission in the cold splitting of $^{252}$Cf and to compare the asymptotic properties with that of alpha emission and experimental values. In ternary fission a third charged fragment is generated in the fission process close to the plane perpendicular to the direction of the two main charged fragments. Ternary fission in which fragments are produced in the ground state (hence maximum kinetic energies) is called cold ternary fission. The theoretical prediction of $^{10}$Be emission was made by Sandulescu et al in 1998 and the first case of neutron less $^{10}$Be ternary spontaneous fission in $^{252}$Cf was reported in the same year [1]. Emission of $^{10}$Be is peculiar in the sense that it is one of the clusters that are not emitted in cluster decay. Further, to definitively establish cold ternary fission or multifragmentation it is necessary to investigate cold neutron less ternary fission with third particle being a heavier cluster such as $^{10}$Be. In hot fission there are too many unknown parameters associated with the initial scission point configurations whereas in cold fission initial conditions of scission are better known because the fragments are of compact shapes at scission, zero excitation energy and high kinetic energy. The kinetic energies and angular distributions of light clusters emitted in cold ternary fission can provide new insight on the fragmentation processes of heavy nuclei.

Theory:

The trajectory calculation of the $^{10}$Be is made by using the Newton’s classical equations of motion. Accordingly the total force acting on the LCP will be

$$F_{aj} = F_{coul.} + F_{nucl.}$$

with

$$a_{x\alpha} = \sum_j \frac{F_{aj}}{m} \left( \frac{X_j - X_\alpha}{r_{aj}} \right)$$

and

$$a_{y\alpha} = \sum_j \frac{F_{aj}}{m} \left( \frac{Y_j - Y_\alpha}{r_{aj}} \right)$$

$$F_{coul.} = \sum_j \frac{e^2 Z_\alpha Z_j}{r_{aj}^2}$$

where

$$r_{aj} = \sqrt{\left( X_\alpha - X_j \right)^2 + \left( Y_\alpha - Y_j \right)^2}$$

Here $j = H, L$ for heavy and light fragments respectively. The nuclear force $F_{nucl}$ is taken as negative gradient of the proximity potential. The position and velocity of $^{10}$Be are determined by the equations,

$$V_{x\alpha}(t) = V_{x\alpha}(t_0) + a_{x\alpha}(t')dt$$

$$V_{y\alpha}(t) = V_{y\alpha}(t_0) + a_{y\alpha}(t')dt$$

$$X_\alpha(t) = X_\alpha(t_0) + V_{x\alpha}(t')dt + \frac{1}{2} a_{x\alpha}(t')dt^2$$

$$Y_\alpha(t) = Y_\alpha(t_0) + V_{y\alpha}(t')dt + \frac{1}{2} a_{y\alpha}(t')dt^2$$

Here finite size of the fragments is taken in to account, since the same will affect the effective tip distance. Since ground state deformation is significant in cold fission we’ve incorporated the deformation of heavy fragments and $^{10}$Be is taken as spherical. The approximate tip distance at scission is calculated by the method suggested by Misicu et al [2] and it is found that the tip distance of the main fragments lies in between 5 to 7 fm. On the other hand for the alpha emission these extreme limits are found to be 6 to 8 fm, which tempted us to conclude that the tip distance decreases as the size if the fragment increases. Nuclear force along with the Coulomb force is also included in our calculation since in these limits of separation nuclear contribution will be significant. Here the proximity potential proposed by Blocki et
al. [3] is used as the nuclear interaction. The Proximity potential \( V_p \) is given as
\[
V_p(z) = \frac{4\gamma}{b} \left[ \frac{C_1C_2}{(C_1+C_2)} \right] \Phi \left( \frac{z}{b} \right)
\]
Here \( \gamma \) nuclear surface tension coefficient, \( \Phi(z/b) \) is the universal proximity potential, \( b \) the width of the diffuseness of nuclear surface and \( z \) is the tip distance. In cold fission the estimation of initial kinetic energy is made by identifying the initial kinetic energy with the zero-energy in the harmonic potential well and can be calculated using the expression
\[
E = \frac{1}{2} \hbar \left( \frac{C}{m} \right).
\]
where \( m \) is the mass of the light charged particle and \( C \) is the stiffness constant which is computed using the formalism given by Misicu et al.[2]. The kinetic energy of the heavy fragments as a function of separation distance \( d \) are calculated as
\[
TKE(d) = TKE_L + TKE_H = Q_{LH} - V_{1LH}(d)
\]
and from the conservation of linear momentum
\[
TKE_i = \left( \frac{A_i}{A_H + A_L} \right) TKE(d)
\]
where \( i = H, L \). The computation of TKE will help us to get an idea about the deformation at scission and the total excitation energy of the fragments.

Results and discussion:
Experimentally the value of kinetic energies for \(^{10}\text{Be}\) emission and alpha emission from \(^{252}\text{Cf}\) is 17.5MeV and 16MeV respectively [4]. We got a final energy of 18.24MeV for \(^{10}\text{Be}\) close to the experimental value for a tip distance of 6fm corresponding to an initial K.E 2.01MeV. The corresponding K.E of the light and heavy fragments is found to be 13.56 and 8.78MeV respectively. The asymptotic angle in the case of \(^{10}\text{Be}\) is found to be nearly 79.51 degree which is nearly same in the case of alpha emission. This nearly identical value of final energy and asymptotic angle increase. Since the mass distributions of the heavier fragments are very similar to those of the cold binary fission of an initial nucleus leading to the same heavy fragments and in the case of \(^{10}\text{Be}\) emission the heavy fragment distribution is like the cold binary fission of \(^{242}\text{Pu}\). The decay energy for such a binary fragmentation will be \( Q_{1H} = Q_i - Q_{LCP} \), where \( Q_i \) is the ternary decay energy of \(^{252}\text{Cf}\) and \( Q_{LCP} \) is 8.71MeV for \(^{10}\text{Be}\).
The Total Kinetic Energy of the heavy fragments is found to be less compared to the alpha emission. The difference is due to the fact that the K.E. of heavy fragment is determined mainly by the Coulomb repulsive force of the heavy fragments and depends on the scission configuration. Even though the energy of the emitted alpha and \(^{10}\text{Be}\) are nearly same, the difference in fragment TKE implies that their scission configurations are not comparable.

![Fig.1: Trajectory of \(^{10}\text{Be}\) for different tip distances](image)

References: