Random matrix ensembles with parity preserving random interactions

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Introduction

Parity ratio of nuclear level densities is an important ingredient in nuclear astrophysical applications [1]. It is now well understood that at moderate excitation energies the parity ratio is not close to unity as assumed in the past. Recently, a Fermi gas model has been developed and used for tabulating parity ratios as a function of excitation energy in large number of nuclei of astrophysical interest. However, a good ab-initio theory for parity ratios is not yet available. With the success of random interaction matrix ensembles (RIMM) [2, 3], one can argue that the ensembles generated by parity preserving random interaction (hereafter called RIMM-PTY) may provide some generic results for parity ratios. In addition, there is also the important recognition in the past few years that random interactions generate regular structures [4]. Then a question is: why ground states of even-even nuclei are always of +ve parity. A simple RIMM-PTY has been identified and analyzed recently [5] to address the question of 'abundance of ground states with positive parity'. Going beyond this, we have constructed more general RIMM-PTY to address the two issues mentioned above and also to examine the form of fixed parity state densities. Here we will give the definition of RIMM-PTY, a method for its construction and some first results.

RIMM-PTY Ensemble

Given $N_+$ number of positive parity single particle (sp) states and similarly $N_-$ number of negative parity states, let us assume for simplicity that the +ve and -ve parity states are degenerate and separated by energy $\Delta$ (see Fig. 1). This defines the one-body part $h(1)$ of the Hamiltonian $H$ with $N = N_+ + N_-$ sp states. The matrix for the two-body part $V(2)$ of $H$ will be a $3 \times 3$ block matrix in two particle spaces as there are three possible ways to generate two particle states with definite parity: (i) both in +ve parity states; (ii) both in -ve parity states; (iii) one in +ve and other in -ve parity states. They will give the matrices $A$, $B$ and $C$ respectively in Fig. 1. For parity preserving interactions only the states (i) and (ii) will be mixed and mixing matrix is $D$ in Fig. 1.

Many particle states for $m$ fermions in the $N$ sp states can obtained by distributing $m_1$ fermions in $N_+$ +ve parity sp states and similarly $m_2$ fermions in the $N_-$ states with $m = m_1 + m_2$. Let us denote each distribution of $m_1$ fermions by $\tilde{m}_1$ and similarly $\tilde{m}_2$. In the many particle basis defined by $(\tilde{m}_1, \tilde{m}_2)$ the $H$ matrix construction reduces to the well known spinless fermion problem [2]. The matrix dimensions ($d_{\pm}$) for +ve parity states follows from the dimensions for all $(m_1, m_2)$ with $m_2$ even and similarly for -ve parity states ($d_{\mp}$) with $m_2$ odd. For example: (i) for $N_+ = N_- = 7$ and $m = 6$, $d_+ = 1484$.

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and $d_- = 1519$; (ii) for $N_+ = N_- = 8$ and $m = 6$, $d_+ = 3976$ and $d_- = 4032$.

The RIMM-PTY is defined by choosing the matrices $A$, $B$ and $C$ to be independent GOE’s with matrix elements variances $v_{A}^{2}$, $v_{B}^{2}$ and $v_{C}^{2}$ respectively. Similarly the matrix elements of the mixing $D$ matrix are chosen to be independent (independent of $A$, $B$ and $C$ matrix elements) zero centered Gaussian variables with variance $v_{D}^{2}$. Without loss of generality we choose $\Delta = 1$ so that all the $v$’s are in $\Delta$ units. Defining the matrix elements variances of the diagonal blocks $A$, $B$ and $C$ to be same (to reduce number of free parameters in RIMM-PTY), we have the RIMM-PTY model defined by two parameters $(\tau, \alpha)$ where

$$ v_{A}^{2} = v_{B}^{2} = v_{C}^{2} = \tau^{2} \text{ and } v_{D}^{2} = \alpha^{2} \tau^{2}. $$

In the limit $\tau^{2} \to \infty$ (with $\alpha^{2} = 1$) the model reduces to the simple model analyzed in [5].

**Results and Discussion**

Firstly, for $N_+ = N_- = m = 6$ system with 200 members, we have verified (using large values for $\tau$ and putting $\alpha = 1$) that $R_+ \sim 20\%$ as given in [5]. Going beyond this, calculations with 100 members for $N_+ = N_- = 7$ and $m = 6$ system are performed using different values for $\tau$ and $\alpha$ parameters defined above. We have numerically studied: (i) percentage of $+ve$ parity ground states $R_+$; (ii) shapes of $+ve$ ($\rho_+$) and $-ve$ parity ($\rho_-$) state densities; (iii) parity ratio $\rho_-/\rho_+$. The first results are as follows. It is seen that with $(\tau, \alpha)$ variation, $R_+$ shows variation and for example: for $\alpha = 0.2$, $R_+$ changes from 100% to 75% as $\tau$ varies from 0.03 to 0.2 and for $\alpha = 2$, $R_+ \sim 100\%$ as $\tau$ varies from 0.03 to 0.2. The state densities $\rho_+$ and $\rho_-$ and the ratio $\rho_-/\rho_+$ are shown in Fig. 2 for some examples. For small $\tau$ values, the densities are multi-modal and as $\tau$ increases to 0.2, they approach Gaussian form. This is verified for $\alpha = 0.2$ to 2. We observe considerable structure in $\rho_-/\rho_+$ for small $\tau$ values. For $\tau \sim 0.2$ and larger, it is seen that $\rho_- \sim \rho_+$ for $E - E_{gs} > \sigma$. Here $gs$ stands for ground state and $\sigma$ is average width over the ensemble for $+ve$ and $-ve$ parity state densities. Similarly in Fig. 1, for $\rho_+$ and $\rho_-$ densities, $E_c$ is energy centroid and $\sigma$ is spectral width for the corresponding densities. We are attempting to derive analytical results for spectral variances to understand $R_+$ variation and also the variation in $\rho_+$, $\rho_-$ and $\rho_-/\rho_+$. Also calculations are being carried out for many different values of $(N_+, N_-, m)$ values and for a much larger range of $(\tau, \alpha)$. It is important to identify the range of $(N_+, N_-, m)$ and $(\tau, \alpha)$ values appropriate for some typical nuclei and then determine $R_+$, $\rho_+$, $\rho_-$ and $\rho_-/\rho_+$ for these system using RIMM-PTY. This exercise is being attempted.

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**References**


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