1. Introduction
Neutron stars are born when a massive star runs out of nuclear fuel and undergoes a supernova explosion in which the core of the star collapses to form a compact object, containing about one and a half times of solar mass inside a sphere of radius about ten kilometers [1]. Due to various perturbations, radial and non-radial waves travel through the star interior, that make the surface oscillate. The non-radial oscillations of neutron stars make them promising sources of detectable gravitational waves. The structure and oscillation modes of neutron stars are governed by the equation of state of nuclear matter. The study of the vibrational motion of neutron stars has become an important tool to constrain the equation of state of nuclear matter. In section (2) we present the formalism of calculating normal mode frequencies of non-radial oscillations, and in (3) the results.

2. Formulation
The bulk motions of nuclear matter can be described in terms of the basic variables of the hydrodynamics, namely, the bulk density, \( \rho \), the mean velocity, \( V_i \), and the stress tensor \( P_{ij} \). The governing equations are written as [2],

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \rho \frac{\partial V_i}{\partial x_i} &= 0, \\
\rho \frac{\partial V_i}{\partial t} + \frac{\partial P_{ik}}{\partial x_k} + \rho \frac{\partial U}{\partial x_i} &= 0, \\
\frac{\partial P_{ij}}{\partial t} + P_{ik} \frac{\partial V_i}{\partial x_k} + P_{jk} \frac{\partial V_j}{\partial x_k} + P_{ij} \frac{\partial V_k}{\partial x_k} &= 0,
\end{align*}
\]

where \( U \) is the internal potential energy. Assuming that the neutron star is a gravitationally bound nuclear matter of uniform density and the local equilibrium and large-scale motions of neutron star are dominated by newtonian gravity, the potential energy \( U \) can be known from Poisson’s equation,

\[
\nabla^2 U = 4\pi G \rho_0 ,
\]

where \( G \) is the gravitational constant and \( \rho_0 \) is the density. The gravitational potential, in equilibrium, can be written for inside and outside region of the star as,

\[
\begin{align*}
U^\text{in}_0 (r) &= \frac{2}{3} \pi G \rho_0 \left( r^2 - 3R^2 \right), \quad r \leq R, \\
U^\text{out}_0 (r) &= \frac{4}{3} \pi G \rho_0 R^3, \quad r > R,
\end{align*}
\]

where \( R \) is the radius of the star. Considering the distribution of the stresses inside the star to be isotropic and the surface to be without stress,

\[
P_{ij} = \delta_{ij} P_0 (r) , \quad P_0 (R) = 0 ,
\]

the local equilibrium pressure, \( P_0 (r) \), can be determined by the equation,

\[
\nabla P_0 (r) = -\rho_0 \nabla U^\text{in}_0 ,
\]

the solution of which gives,

\[
P_0 (r) = -\frac{2}{3} \pi G \rho_0 \frac{r^2}{\rho_0} + P_F ,
\]

where \( P_F \) is the pressure at core. Radius of neutron star can be obtained from (7),

\[
P_0 (R) = 0, \quad \Rightarrow \quad R = \frac{1}{\rho_0} \sqrt{\frac{3P_F}{2\pi G}} ,
\]

consequently mass of the star can be found as,

\[
M = \frac{4}{3} \pi R^3 \rho_0 = \frac{4\pi}{3\rho_0} \left( \frac{3P_F}{2\pi G} \right)^{\frac{3}{2}} ,
\]

where the core pressure, \( P_F \), can be known from the equation of state given the average

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density $\rho_0$ which is the only parameter of this model.

To calculate the fundamental nodes of oscillations of highly incompressible nuclear matter, variational method [3] can be applied to rewrite the equations (1) - (4) as,

$$\frac{\partial \delta V_i}{\partial x_i} = 0, \quad (12)$$

$$\rho_0 \frac{\partial \delta V_i}{\partial t} + \frac{\partial \delta P_{jk}}{\partial x_k} + \rho_0 \frac{\partial \delta U}{\partial x_i} = 0, \quad (13)$$

$$\frac{\partial \delta P_{ij}}{\partial t} + \frac{\partial \delta V_i}{\partial x_j} \frac{\partial P_0}{\partial x_j} + \frac{\partial \delta V_i}{\partial x_j} \frac{\partial P_0}{\partial x_j} = 0, \quad (14)$$

$$\nabla^2 \delta U = 0, \quad (15)$$

where $\delta V_i$ is the perturbed velocity of collective flow and $\delta P_{ij}$ is small fluctuation in the stress tensor. Multiplying (13) by $\delta V_i$ and integrating over the volume gives the energy balance equation as,

$$\frac{\partial}{\partial t} \int \left( \frac{\rho_0 V_i^2}{2} \right) d\tau = \int \delta P_{ij} \frac{\partial \delta V_i}{\partial x_j} d\tau$$

$$- \oint_s [\rho_0 \delta U \delta V_i + \delta P_{ij} \delta V_j] d\sigma, \quad (16)$$

We represent $\delta V_i$ and $\delta U$ in the form,

$$\delta V_i = \xi^L_i(\vec{r}) \dot{\alpha}_L(t), \quad \delta U = \phi^L(\vec{r}) \alpha_L(t), \quad (17)$$

where $L$ denotes the multipole order. The amplitude, $\alpha_L$, gives spheroidal deformations of the star surface, and $\xi^L_i(\vec{r})$ is the instantaneous displacements given by

$$\xi^L_i = \frac{1}{L! R^{L-2}} \frac{d}{dx_i} r^{L-1} P_L(\cos \theta), \quad (18)$$

and the function $\phi_L$ on the star’s surface has the form [3],

$$\phi^L = -\frac{4\pi G\rho_0 R^2}{2L+1} P_L(\cos \theta). \quad (19)$$

Inserting (17) into (14), the fluctuations in the stress tensor can be written as,

$$\delta P_{ij}(\vec{r}, t) = -\left[ P_0 \left( \frac{\partial \xi^L_i}{\partial x_j} + \frac{\partial \xi^L_j}{\partial x_i} \right) \right. \frac{\partial P_0}{\partial x_k} \left. \right] \alpha_L \cdot (20)$$

Substituting (17) and (20) in (16), we get the equation of normal vibrations,

$$M_L \ddot{\alpha}_L(t) + K_L \alpha_L(t) = 0, \quad (21)$$

where $M_L$ and $K_L$ are inertia and stiffness parameters respectively. The fundamental frequencies, $\omega_L^2 = K_L/M_L$, of the non-radial vibrations of a neutron star can be computed as,

$$\omega_L^2 = \omega_2^2 \frac{2(2L+1)(L-1)}{2L+1} - \omega_2^2 \frac{4L^2 - 1}{2L - 1}, \quad (22)$$

where $\omega_2^2 = P_F/(\rho_0 R^2)$ and $\omega_2^2 = 4\pi G\rho_0/3$. The basic frequency of quantum-elastic vibrations, $\omega_F$, depends on incompressibility, density and radius of the star, whereas and gravitation-elastic vibrations $\omega_G$ depends on the density alone.

### 3. Results

We take the equation of state of nuclear matter from [4] where it is calculated in the framework of non-relativistic Brueckner formalism using Bonn potential and medium modification of meson parameters. In the following table we present numerical estimates for the mass $M$ (normalized to the solar mass, $M_{\odot}$), the radius $R$ (in Km) of a neutron star and frequencies (in $10^4$ Hz) of quadrupole($\omega_2$), octupole($\omega_3$) and hexadecapole($\omega_4$) vibrations corresponding to the average density $\rho_0$ (in fractions of the normal nuclear density $\rho_N$).

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<th>$\rho_0$</th>
<th>$M$</th>
<th>$R$</th>
<th>$\omega_2$</th>
<th>$\omega_3$</th>
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### References


