Calibration of nuclear matter parameters in an effective chiral model

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Introduction

Recently, there have been some efforts to generate new parameters of the model [1] to extend its applicability to nuclear matter studies. With similar motivation, here we evaluate the parameters of an effective sigma model [2] and analyze the equation of state so obtained. This shall enable us to study and correlate some fundamental properties of matter such as nucleon effective mass and nuclear incompressibility, both of which are not precisely known. Nuclear equation of state is the primary input that goes in the determination of the structural properties of the neutron star, such as the mass and the radius. The extreme of densities prevailing in the core of these stars render stable exotics in the form of hyperons and quarks, the composition and concentration of which are EoS dependant. In the present work, we look into these aspects and analyze the correlations between properties such as nucleon effective mass ($m^*$), the nuclear incompressibility ($K$), and the resulting equation of state of dense matter.

The Model and the Methodology

The present model embodies dynamically generated mass of the vector meson, which also plays role in regulating the the nucleon effective mass, in addition to the higher orders of the scalar field interaction. The effective Lagrangian of the model includes the pseudoscalar meson $\pi$, the scalar meson $\sigma$, the vector meson $\omega$ and the iso-vector $\rho$-meson. The interaction of the scalar and the pseudoscalar mesons with the vector boson generates a dynamical mass for the vector bosons through spontaneous breaking of the chiral symmetry and the scalar field attain a vacuum expectation value $\sigma_0$. Then the mass of the nucleon ($m$), the scalar ($m_\sigma$) and the vector meson mass ($m_\omega$), are related to the scalar condensate.

The meson field equations are solved self-consistently at a fixed baryon density and the corresponding energy density and pressure is calculated. however, we need to evaluate the parameters of the model (the coupling constants $C_\sigma, C_\omega, C_\rho$ and the higher order scalar field constants $B$ and $C$) that satisfy nuclear matter saturation properties. To evaluate them, we follow the standard procedure of calibrating the parameters with respect to the known properties at saturation density. At the standard state $\rho_B = \rho_0 = 0.153 fm^{-3}$, the nuclear matter saturation density for symmetric nuclear matter, the energy per particle is $e(\rho_0) = \varepsilon/\rho_0 - m = a_1 \approx -16$ MeV. Further, the equilibrium condition requires that $P(\rho_0,0) = 0$ i.e.,

$$\varepsilon = \varepsilon_k + \varepsilon_\sigma + \varepsilon_\omega = -16 MeV$$ \hspace{1cm} (1)

$$P = -\varepsilon + \rho_B \frac{\partial \varepsilon}{\partial \rho_B} = \frac{1}{3} \varepsilon_k - \frac{1}{3} m^* \rho_S - \varepsilon_\sigma + \varepsilon_\omega = 0$$ \hspace{1cm} (2)

With the aforesaid methodology, we evaluate the nuclear matter parameters at different saturation density in the mean field approach. The parameters with variation in saturation density would project the correlations between the fundamental properties of matter
FIG. 1: On the left panel, the variation of the scalar, the vector and the iso-vector coupling constant are displayed and on the right panel, the nucleon effective mass, the incompressibility and the mass of the sigma meson is shown with varying saturation density corresponding to both the linear (Lσ) and the non-linear (NLσ) scalar field interactions.

Results and Discussion

The spontaneous breaking of chiral symmetry lends mass to the hadrons and relates them to the vacuum expectation value (VEV) of the scalar field (σ₀). The VEV of the scalar field which has a minimum potential at $\sigma = f_\pi$ is found to be directly related to the vector coupling constant $C_\omega$ through $\sigma_0 = f_\pi = m_\omega / g_\omega = 1 / \sqrt{C_\omega}$. Similarly, the mass of the scalar meson can be given by $m_\sigma = m \sqrt{C_\omega} / \sqrt{C_\rho}$. We also demand that the matter is bounded from below, i.e., the coefficient ‘$C$’ in the quartic scalar field term remains positive and the resulting energy per particle for symmetric nuclear matter is $\approx -16MeV$.

It is to be noted that the nonlinearity in the scalar field provides freedom to fine tune the parameters so that they are compatible with the pion decay constant. However, the difference between the linear and the nonlinear scalar field interaction seems to wash out at higher saturation density. The present analysis projects the interlink between the fundamental properties of matter such as the nucleon effective mass, the nuclear incompressibility and the sigma meson mass. The experimental value of the pion decay constant serves as a stern constraint on the vector coupling and in turn regulates the nucleon effective mass in the present model. Within the acceptable range of saturation density $\rho_0 = 0.155 \pm 0.005 fm^{-3}$ the mass of the sigma meson comes out in the range $m_\sigma = 546 \pm 10 MeV$, which agrees well with recent experimental bounds. The present model predicts EoS with high incompressibility is within the upper range of the bound from the HIC data, and is also compatible with the DBHF prediction. The resulting high value of nucleon effective mass in the present model is a consequence of the dominant repulsive force in matter at high density, which may have interesting implications in the astrophysical context such as the modelling of neutron stars where the matter density is speculated to be in the range $(3 - 10)\rho_0$ [3]. Overall, the model seems to provide a unified description of nuclear matter aspects, however the high incompressibility, although acceptable in the astrophysical domain needs to be brought down to enhance the applicability of the model in further studies. Work is in progress in this direction [4].

References